Day 155 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

60 fish in a lake are caught, tagged and put back.
Later, a second sample of 50 fish is caught. 3 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 3 out of 50

This should match the proportion tagged in the whole population: $\frac{3}{50} = \frac{60}{N}$

N = 60 × 50 ÷ 3 = 1000

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 1000 (b) The population stays the same between samples

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x + 10)°
Angle ABC = (x + 16)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x + 10) + 2(x + 16) = 180
6x + 42 = 180, so 6x = 138
x = 23
(b) Angle BAC = 4 × 23 + 10 = 102°
Answer: (a) x = 23 (b) 102°

3 Arithmetic sequences

The 2nd term of an arithmetic sequence is 24.
The 6th term of the sequence is 32.
(a) Find an expression for the nth term of the sequence.
(b) Is 97 a term of the sequence? You must show your working.

(a) Common difference = (32 − 24) ÷ (6 − 2) = 2
nth term = 2n + c. For n = 2: 2 × 2 + c = 24, so c = 20
nth term = 2n + 20

(b) Solve 2n + 20 = 97: 2n = 77, so n = $\frac{77}{2}$

n is not a whole number, so 97 is not a term
Answer: (a) 2n + 20 (b) No

4 Fraction problems

A bag contains sweets.
$\frac{1}{3}$ of the sweets are orange and $\frac{1}{4}$ of the sweets are lemon.
The rest of the sweets, 30 of them, are strawberry.
How many sweets are in the bag?

Fraction that are strawberry = $1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$

$\frac{5}{12}$ of the total is 30

Total = $30 \div \frac{5}{12} = 72$

Answer: 72

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5 Inverse proportion in context

10 painters can paint a hall in 20 days.
After 5 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 10 × 20 = 200 painter-days
Work done = 10 × 5 = 50, so work left = 150
Workers left = 10 − 5 = 5
Time = 150 ÷ 5 = 30 days
Answer: 30 days

6 Prime factors and indices

Write $12^4 \times 9^3$ as a product of powers of prime numbers.

$12 = 2^2 \times 3$, $9 = 3^2$
$12^4 = 2^8 \times 3^4$
$9^3 = 3^6$
Add the powers of each prime: $2^8 \times 3^{10}$
Answer: $2^8 \times 3^{10}$

7 Equations in disguise

Solve $x^4 - 26x^2 + 25 = 0$

Let $u = x^2$: $u^2 - 26u + 25 = 0$
Factorise: $(u - 1)(u - 25) = 0$, so $u = 1$ or $u = 25$
$x^2 = 1$ or $x^2 = 25$
$x = \pm 1$ or $x = \pm 5$
Answer: $x = -5, -1, 1, 5$

8 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 2, 16, 36, 36
Mid-points: 15, 30, 55, 90
$\Sigma fm = 2{\times}15 + 16{\times}30 + 36{\times}55 + 36{\times}90 = 5730$, $\Sigma f = 90$
Mean = 5730 ÷ 90 = 63.7
Answer: 63.7

9 Tangent to a circle: proof

Show that the line $y = x - 10$ is a tangent to the circle $x^2 + y^2 = 50$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x - 10$ into $x^2 + y^2 = 50$: $x^2 + (x - 10)^2 = 50$
Expand: $2x^2 - 20x + 50 = 0$
$b^2 - 4ac = (-20)^2 - 4 \times 2 \times 50 = 400 - 400 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-(-20)}{2 \times 2} = 5$, and $y = 5 - 10 = -5$. The point is $(5, -5)$
Answer: $(5, -5)$

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10 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the opposite side BC, so use tan 60°

$\tan 60° = \sqrt{3}$

$x = 7 \div \tan 60° = 7 \div \sqrt{3}$

$x = \frac{7\sqrt{3}}{3}$ (after rationalising the denominator)

Answer: x = $\frac{7\sqrt{3}}{3}$