Day 154 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

80 fish in a lake are caught, tagged and put back.
Later, a second sample of 120 fish is caught. 6 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 6 out of 120

This should match the proportion tagged in the whole population: $\frac{6}{120} = \frac{80}{N}$

N = 80 × 120 ÷ 6 = 1600

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 1600 (b) The population stays the same between samples

2 Solving quadratics by factorising

Solve $12x^2 - 3 = 5x$

Rearrange so that one side is 0: $12x^2 - 5x - 3 = 0$
Factorise: $(4x - 3)(3x + 1) = 0$
$4x - 3 = 0$ or $3x + 1 = 0$
$x = \frac{3}{4}$ or $x = -\frac{1}{3}$

Answer: $x = -\frac{1}{3}$ or $x = \frac{3}{4}$

3 Fractions

Work out $(1\frac{1}{2} + 2\frac{1}{4}) \div 2\frac{1}{2}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{3}{2} + \frac{9}{4} = \frac{15}{4}$
$\frac{15}{4} \div \frac{5}{2} = \frac{15}{4} \times \frac{2}{5} = \frac{3}{2}$
$= 1\frac{1}{2}$

Answer: $1\frac{1}{2}$

4 Comparing box plots

The box plots show the times taken by two groups, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 75 − 35 = 40
IQR of B = 55 − 35 = 20
(b) Group B has the lower median (40 compared with 55), so on average Group B was faster
Group B has the smaller interquartile range (20 compared with 40), so its times are more consistent
Answer: (a) A: 40, B: 20 (b) B has the lower median, B has the smaller IQR

5 Substitution

Work out the value of $\frac{2m - n}{k}$
when $m = -4$, $n = -3$ and $k = 3$.

Substitute the values: $\frac{2 \times (-4) - (-3)}{3}$
$= \frac{-5}{3}$
$= -\frac{5}{3}$

Answer: $-\frac{5}{3}$

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6 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 1.5x + 10.
(a) Estimate y when x = 9.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) y = 1.5 × 9 + 10 = 23.5
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 23.5 (b) x = 15 is outside the range of the data

7 Combined transformations

The point P(2, 6) is reflected in the x-axis.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (2, −6)
After the second transformation: P'' = (−6, −2)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−y, −x)
The single mapping is (x, y) → (−y, −x)
This is a reflection in the line y = −x
Answer: (a) (−6, −2) (b) Reflection in the line y = −x

8 HCF and LCM

The HCF of two numbers is 12 and their LCM is 72.
One of the numbers is 36.
Work out the other number.

HCF × LCM = product of the two numbers
12 × 72 = 864
Other number = 864 ÷ 36 = 24
Answer: 24

9 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 41°.
B is a point on the same line, 80 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 16°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 41° = \frac{h}{x}$, so x = h ÷ tan 41°

From B: $\tan 16° = \frac{h}{x + 80}$, so x + 80 = h ÷ tan 16°

Subtract: 80 = h ÷ tan 16° − h ÷ tan 41° = h(3.4874 − 1.1504)
h = 80 ÷ 2.3370 = 34.2 m (1 d.p.)
Answer: 34.2 m

10 Bounds and accuracy

a = 0.327 and b = 3.45, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

0.3265 ≤ a < 0.3275; 3.445 ≤ b < 3.455
Lower bound = 0.3265 ÷ 3.455 = 0.094501...
Upper bound = 0.3275 ÷ 3.445 = 0.095065...
Both bounds round to 0.095 to 2 significant figures, but not to 3 significant figures
Answer: 0.095 (2 s.f.)