Day 153 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x + 15)°
Angle PQR = (2x − 10)°
Angle PRS = (4x + 37)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 37 = 6x + 5
2x = 32
x = 16
(b) Angle PRS = 101° so angle PRQ = 180° − 101° = 79°
Answer: (a) x = 16 (b) 79°

2 Time zones

A flight leaves London at 12:40 (London time).
It takes 10 hours 40 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 12:40 + 10 h 40 min = 23:20
New York is 5 hours behind London, so subtract 5 hours
18:20
Answer: 18:20 the same day

3 Comparing costs

Sparkle Cleaners has a call-out fee of £3.50 plus £6 per hour.
Fresh Start has a call-out fee of £56 plus £2.50 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 20 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when 3.50 + 6n = 56 + 2.50n
3.50n = 52.50, so n = 15
(b) Sparkle Cleaners: £3.50 + £6 × 20 = £123.50
Fresh Start: £56 + £2.50 × 20 = £106
Fresh Start is cheaper by £17.50
Answer: (a) 15 (b) Fresh Start, by £17.50

4 Area and quadratic equations

The diagram shows a rectangle of length (x + 4) cm, width (2x + 3) cm and area 228 cm^2.
(a) Show that $2x^2 + 11x - 216 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) (x + 4)(2x + 3) = 228
Expand: $2x^2 + 11x + 12 = 228$, so $2x^2 + 11x - 216 = 0$
(b) Factorise: (x − 8)(2x + 27) = 0
x = 8 ($x = -\frac{27}{2}$ is not possible as a length would be negative)
Length = 12 cm and width = 19 cm
Diagonal = $\sqrt{12^2 + 19^2} = \sqrt{505}$ = 22.5 cm (3 s.f.)
Answer: (b) 22.5 cm

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5 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 55 − 35 = 20
IQR of B = 60 − 25 = 35
(b) Class B has the higher median (55 compared with 50), so on average Class B scored higher
Class A has the smaller interquartile range (20 compared with 35), so its marks are more consistent
Answer: (a) A: 20, B: 35 (b) B has the higher median, A has the smaller IQR

6 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 10 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 20 cm

Area of the square = $\frac{1}{2}$ × diagonal × diagonal = $\frac{1}{2}$ × 20 × 20 = 200

Area of circle = $\pi \times 10^2 = 100\pi$

Shaded area = $100\pi - 200$ cm^2

Answer: $(100\pi - 200)$ cm^2

7 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{1}{20}$.

Events on different occasions are independent.
Work out the smallest number of mornings needed so that the probability of this happening at least once is greater than 0.5. (Calculator)

P(at least once in n mornings) = $1 - \frac{19}{20}^n$

n = 13: $1 - \frac{19}{20}^{13} = 0.4867$

n = 14: $1 - \frac{19}{20}^{14} = 0.5123$

0.5123 is the first value above 0.5
Answer: 14

8 Estimating the median from grouped data

The table gives information about 72 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 27, 42, 46, 57, 72
The median is the value at position 72 ÷ 2 = 36
It is in the class 30 to 40, with 27 values before it, so we need 9 more of the 15 values in this class

Median ≈ 30 + $\frac{9}{15}$ × 10 = 36

Answer: 36

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 54 ÷ k is a square number.

$54 = 2 \times 3^3$
For a square number every power must be even
Divide by 2 × 3 to get 3^2
k = 6
Answer: k = 6

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10 Functions that are their own inverse

$f(x) = \frac{-3x - 2}{3x + 3}$ where $x \neq -1$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (3x + 3)

$ff(x) = \frac{(-3)(-3x - 2) + (-2)(3x + 3)}{3(-3x - 2) - (-3)(3x + 3)}$

Top $= 9x + 6 - 6x - 6 = 3x$

Bottom $= -9x - 6 + 9x + 9 = 3$

$ff(x) = \frac{3x}{3} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{-3x - 2}{3x + 3}$

Answer: (b) $f^{-1}(x) = \frac{-3x - 2}{3x + 3}$