Day 152 **S Tier 10-a-day: Model Solutions**

1 Fractions removed from a ratio

The ratio of men to women people at a meeting is 7 : 6.

Then $\frac{2}{3}$ of the men and $\frac{1}{2}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{1}{3} \times 7 = \frac{7}{3}$ of the men part, $\frac{1}{2} \times 6 = 3$ of the women part

$\frac{7}{3} : 3$

Multiply by 3: 7 : 9

Answer: 7 : 9

2 Surds

Given that $\sqrt{125} - \sqrt{20} = \sqrt{n}$,
find the value of n.

$\sqrt{125} = 5\sqrt{5}$ and $\sqrt{20} = 2\sqrt{5}$

$5\sqrt{5} - 2\sqrt{5} = 3\sqrt{5}$

$3\sqrt{5} = \sqrt{45}$, so n = 45

Answer: n = 45

3 Arc length and sector area

A sector of a circle has radius 15 cm and angle 90°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{90}{360}$

Arc length = $\frac{90}{360} \times 2 \times \pi \times 15 = \frac{15}{2}\pi$ cm

(b) Area = $\frac{90}{360} \times \pi \times 15^2 = \frac{225}{4}\pi$ cm^2

Answer: (a) $\frac{15}{2}\pi$ cm (b) $\frac{225}{4}\pi$ cm^2

4 Capture and recapture

60 fish in a lake are caught, tagged and put back.
Later, a second sample of 100 fish is caught. 12 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 12 out of 100

This should match the proportion tagged in the whole population: $\frac{12}{100} = \frac{60}{N}$

N = 60 × 100 ÷ 12 = 500

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 500 (b) The population stays the same between samples

5 Transformations of graphs

The point P(−4, 6) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x) + 3 (b) y = f(−x)

(a) y = f(x) + 3 is a translation of 3 up: (−4, 9)

(b) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (4, 6)

Answer: (a) (−4, 9) (b) (4, 6)

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6 Exponential growth and decay

A radioactive sample has a mass of 960 g. Its mass halves every 3 hours.
(a) Work out the mass after 12 hours.
(b) After how many hours will the mass first be less than 25 g?

(a) 12 hours is 4 half-lives: $960 \div 2^4 = 60$ g
(b) Halve repeatedly: 960, 480, 240, 120, 60, 30, 15
15 is the first value below 25, after 6 half-lives, so $6 \times 3 = 18$ hours
Answer: (a) 60 g (b) 18 hours

7 Rationalising the denominator

Write $\frac{6}{5-\sqrt{7}}$ in the form $\frac{p+q\sqrt{7}}{r}$
where p, q and r are integers and $r > 1$,
and the fraction is in its simplest form.

Multiply top and bottom by $(5+\sqrt{7})$:
$\frac{6}{5-\sqrt{7}} = \frac{6(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})}$
Denominator: $25 - 7 = 18$
$= \frac{5+\sqrt{7}}{3}$
Answer: $\frac{5+\sqrt{7}}{3}$

8 Exact lengths and areas

A rectangle has area $(21 + 14\sqrt{2})$ cm^2 and length $(3 + \sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{21+14\sqrt{2}}{3+\sqrt{2}}$
Multiply top and bottom by $(3-\sqrt{2})$:
Numerator: $(21 + 14\sqrt{2})(3-\sqrt{2}) = 35 + 21\sqrt{2}$
Denominator: $9 - 2 = 7$
Width = $5 + 3\sqrt{2}$
Answer: $5 + 3\sqrt{2}$ cm

9 Equations in disguise

Solve $x - 6\sqrt{x} + 8 = 0$

Let $u = \sqrt{x}$: $u^2 - 6u + 8 = 0$
Factorise: $(u-4)(u-2) = 0$, so $u = 4$ or $u = 2$
$\sqrt{x} = 4$ or $\sqrt{x} = 2$
$x = 16$ or $x = 4$
Answer: x = 4 or x = 16

10 Shaded areas: exact answers

The diagram shows an equilateral triangle inside a circle of radius 10 cm.
The three vertices of the triangle lie on the circle.
Work out the area of the shaded region. Give your answer in the form $a\pi - b\sqrt{3}$.

Split the triangle into 3 isosceles triangles with two sides of 10 cm and an angle of 120° between them
Area of the triangle = $3 \times \frac{1}{2} \times 10 \times 10 \times \sin 120° = 150 \times \frac{\sqrt{3}}{2} = 75\sqrt{3}$
Area of circle = $\pi \times 10^2 = 100\pi$
Shaded area = $100\pi - 75\sqrt{3}$ cm^2
Answer: $(100\pi - 75\sqrt{3})$ cm^2