Day 151 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\sqrt{y^2 - 4xz}$
when x = −4, y = −4 and z = 3.

Substitute the values: $\sqrt{(-4)^2 - 4 \times (-4) \times 3}$
$= \sqrt{64}$
$= 8$
Answer: 8

2 Trigonometric graphs

The diagram shows the graph of y = a sin x + c.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when x = 330°.

(a) The maximum value is 5 and the minimum value is 1
a = (5 − (1)) ÷ 2 = 2
c = (5 + (1)) ÷ 2 = 3

(b) y = 2 sin 330° + 3, and $\sin 330° = -\frac{1}{2}$

y = 2
Answer: (a) a = 2, c = 3 (b) y = 2

3 Tests of divisibility

The 4-digit number 70k4 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 7 + 0 + 4 + k = 11 + k
This must be a multiple of 9, so k = 7
Answer: k = 7

4 Volume and surface area of composite solids

A solid is made from a cylinder of height 11 cm and a hemisphere. Both have radius 3 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cylinder: $\pi \times 3^2 \times 11 = 99\pi$

Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 = 18\pi$

Total = $99\pi + 18\pi = 117\pi$ cm^3
Answer: 117π cm^3

5 Average speed

Fatima drives 50 km at an average speed of 25 km/h.
The drive then continues for another 55 km at an average speed of 55 km/h.
Work out the average speed for the whole journey.

Time for first part = 50 ÷ 25 = 2 hours
Time for second part = 55 ÷ 55 = 1 hour
Total distance = 105 km, total time = 3 hours
Average speed = 105 ÷ 3 = 35 km/h
Answer: 35 km/h

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6 HCF and LCM

(a) Write 648 and 6 as products of prime factors.
(b) Find the HCF and the LCM of 648 and 6.
Give your answers as numbers.

(a) $648 = 2^3 \times 3^4$ and $6 = 2 \times 3$
(b) HCF $= 2 \times 3 = 6$
LCM $= 2^3 \times 3^4 = 648$
Answer: (a) $2^3 \times 3^4$; 2×3 (b) HCF = 6, LCM = 648

7 Rates of work

Pipe A can fill an empty pool in 15 minutes.
Pipe B can fill an empty pool in 30 minutes.
Both are turned on at the same time.
How long do they take to fill an empty pool?

In 1 minute: Pipe A does $\frac{1}{15}$ and Pipe B does $\frac{1}{30}$

Together: $\frac{1}{15} + \frac{1}{30} = \frac{45}{450}$

Time $= \frac{450}{45} = 10$ minutes

Answer: 10 minutes

8 Equations with algebraic fractions

Solve $\frac{3}{x+2} - \frac{6}{x+3} = 2$

Multiply every term by $(x + 2)(x + 3)$: $3(x + 3) - 6(x + 2) = 2 \times (x + 2)(x + 3)$
Expand and rearrange so that one side is 0: $2x^2 + 13x + 15 = 0$
Factorise: $(x + 5)(2x + 3) = 0$
$x = -5$ or $x = -\frac{3}{2}$

Answer: $x = -5$ or $x = -\frac{3}{2}$

9 Fractional indices

Given that $x > 0$, solve $x^{\frac{3}{2}} = 1728$

Raise both sides to the power $\frac{2}{3}$

$x = (1728)^{\frac{2}{3}}$
$x = 144$
Answer: x = 144

10 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 3**a** + 5**b**, OQ = 5**a** + 9**b** and OR = 6**a** + 11**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 2**a** + 4**b**
QR = OR − OQ = **a** + 2**b**

QR $= \frac{1}{2}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 1
Answer: (b) 2 : 1