Day 150 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $m^2 - 3nk$
when $m = -4$, $n = -2$ and $k = 1$.

Substitute the values: $(-4)^2 - 3 \times (-2) \times 1$
$= 16 - (-6)$
$= 22$
Answer: 22

2 Fractional indices

Given that $x > 0$, solve $x^{\frac{5}{4}} = 243$

Raise both sides to the power $\frac{4}{5}$

$x = (243)^{\frac{4}{5}}$
$x = 81$
Answer: x = 81

3 Arithmetic sequences

The 6th term of an arithmetic sequence is 33.
The 13th term of the sequence is 54.
(a) Find an expression for the nth term of the sequence.
(b) Is 118 a term of the sequence? You must show your working.

(a) Common difference $= (54 - 33) \div (13 - 6) = 3$
nth term $= 3n + c$. For $n = 6$: $3 \times 6 + c = 33$, so $c = 15$
nth term $= 3n + 15$

(b) Solve $3n + 15 = 118$: $3n = 103$, so $n = \frac{103}{3}$

n is not a whole number, so 118 is not a term
Answer: (a) 3n + 15 (b) No

4 Standard form

Work out $(5.6 \times 10^{-6}) \div (2 \times 10^{-3})$
Give your answer in standard form.

$5.6 \div 2 = 2.8$
$10^{-6} \div 10^{-3} = 10^{-3}$
$= 2.8 \times 10^{-3}$
Answer: 2.8×10^{-3}

5 Fractions removed from a ratio

The ratio of red to blue marbles in a jar is 3 : 7.
Then $\frac{1}{6}$ of the red and $\frac{2}{3}$ of the blue are taken out.
Work out the ratio of red to blue now.

Left: $\frac{5}{6} \times 3 = \frac{5}{2}$ of the red part, $\frac{1}{3} \times 7 = \frac{7}{3}$ of the blue part

$\frac{5}{2} : \frac{7}{3}$
Multiply by 6: 15 : 14
Answer: 15 : 14

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6 Gradient of a chord

A and B are points on the curve $y = x^3 - 4x$.
The x coordinate of A is -3 and the x coordinate of B is 0.
Work out the gradient of the line AB.

At A: $y = -15$. At B: $y = 0$
A is $(-3, -15)$ and B is $(0, 0)$
Gradient $= \frac{0-(-15)}{0-(-3)} = \frac{15}{3} = 5$

Answer: 5

7 Pythagoras with algebra

ABC is a right-angled triangle with sides $(2x + 5)$ cm, $(2x - 2)$ cm and $(2x + 7)$ cm. The longest side is $(2x + 7)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(2x + 5)^2 + (2x - 2)^2 = (2x + 7)^2$
Expand and collect terms: $4x^2 - 16x - 20 = 0$
Factorise: $4(x - 5)(x + 1) = 0$
$x = 5$ or $x = -1$, and x must be positive, so $x = 5$
(b) The two shorter sides are 15 cm and 8 cm
Area $= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

Answer: (a) x = 5 (b) 60 cm^2

8 Reverse percentages

The price of a sofa is increased by 25%.
Later the new price is reduced by 10% in a sale.
The sale price is £652.50.
Work out the price before the increase. (Calculator)

Multipliers: 1.25 and 0.9
Overall multiplier $= 1.25 \times 0.9 = 1.125$
Original $\times 1.125 =$ £652.50
Original $=$ £652.50 $\div 1.125 =$ £580
Answer: £580

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 15,000 ÷ k is a square number.

$15{,}000 = 2^3 \times 3 \times 5^4$
For a square number every power must be even
Divide by 2×3 to get $2^2 \times 5^4$
$k = 6$
Answer: k = 6

10 Exact trigonometric values

Work out the exact value of $\sin 30° \div \tan 60°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\sin 30° \div \tan 60° = \frac{1}{2} \div \sqrt{3}$

$= \frac{\sqrt{3}}{6}$

Answer: $\frac{\sqrt{3}}{6}$