Day 15 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

$\frac{9}{17}$, 0.79, 0.56, $\frac{3}{7}$

Convert each to a decimal: $\frac{9}{17}$ = 0.529, 0.79 = 0.79, 0.56 = 0.56, $\frac{3}{7}$ = 0.429

In order: $\frac{3}{7}$, $\frac{9}{17}$, 0.56, 0.79

Answer: $\frac{3}{7}$, $\frac{9}{17}$, 0.56, 0.79

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 68°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 68°
Angle AOB = 180 − 2 × 68 = 44°
(b) The angle at the circumference is half the angle at the centre: 44 ÷ 2 = 22°
Answer: (a) 44° (b) 22°

3 Percentage change

The population of a village decreased by 40% in 2023.
In 2024 the population decreased by a further 40%.
Work out the overall percentage decrease over the two years.

Multipliers: 0.6 and 0.6
Overall multiplier = 0.6 × 0.6 = 0.36
0.36 − 1 = −0.64, so an overall decrease of 64%
Answer: 64% decrease

4 Pressure

A concrete slab in the shape of a cuboid weighs 690 N.
Its smallest face has area 150 cm^2. Its largest face has area 1,200 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 1,200 cm^2 = 0.12 m^2
(a) Greatest pressure is on the smallest face: 690 ÷ 0.015 = 46,000 N/m^2
(b) Least pressure is on the largest face: 690 ÷ 0.12 = 5,750 N/m^2
Answer: (a) 46,000 N/m^2 (b) 5,750 N/m^2

5 Column vectors

$\mathbf{a} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.
Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} -4 - 4k \\ 3 + 3k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{-4 - 4k}{5} = \frac{3 + 3k}{5}$

5(−4 − 4k) = 5(3 + 3k)
k × (−35) = 35
k = −1
Answer: $k = -1$

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6 Geometric sequences

The 2nd term of a geometric sequence is 192.
The 4th term of the sequence is 1728.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 1728 \div 192 = 9$
$r = \sqrt{9} = 3$ (positive, since all terms are positive)
(b) The 2nd term is $a \times r^1$: $a \times 3 = 192$
$a = 192 \div 3 = 64$
Answer: (a) r = 3 (b) 64

7 Percentage change and proportion

The time taken for a journey is inversely proportional to the speed.
The speed is increased by 300%.
Work out the percentage change in the time taken for a journey.

The speed is multiplied by 4
The multiplier for the time taken for a journey is $1 \div 4 = 0.25$
0.25 is a decrease of 75%
Answer: 75% decrease

8 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 78°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: $360 - 90 - 90 - 78 = 102°$
(b) The angle at the circumference is half the angle at the centre: $102 \div 2 = 51°$
Answer: (a) 102° (b) 51°

9 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -5$ and $x = 1$, so $y = a(x + 5)(x - 1)$
When $x = 0$, $y = -5$: $a \times 5 \times (-1) = -5$, so $a = 1$
$y = (x + 5)(x - 1)$
$y = x^2 + 4x - 5$
Answer: $y = x^2 + 4x - 5$

10 Ratio and expressions

x : y = 7 : 2
Work out the ratio (5x − 5y) : (1x + 5y)
Give your answer in its simplest form.

x : y = 7 : 2, so let x = 7t and y = 2t
5(7t) − 5(2t) = 25t
1(7t) + 5(2t) = 17t
25t : 17t = 25 : 17
Answer: 25 : 17