Day 149 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $3x^2 - 4x - 7$

$3 \times (-7) = -21$. Two numbers that multiply to -21 and add to -4 are 3 and -7

Split the middle term: $3x^2 + 3x - 7x - 7$

Factorise in pairs: $3x(x + 1) - 7(x + 1)$

$= (3x - 7)(x + 1)$

Answer: $(3x - 7)(x + 1)$

2 Mean of a changing set

The mean of 7 numbers is 149.

One number is removed and the mean of the remaining 6 numbers is 169.

Work out the number that was removed.

Total of the 7 numbers = $7 \times 149 = 1043$

Total of the 6 remaining = $6 \times 169 = 1014$

Number removed = $1043 - 1014 = 29$

Answer: 29

3 Rational and irrational numbers

Here are six numbers.

$\sqrt{0.25}$, $\sqrt{50}$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $\sqrt{\frac{9}{16}}$, $(\sqrt{3})^2$, $3 + \sqrt{12}$

Write down the rational numbers.

Simplify each number:

$\sqrt{0.25} = 0.5$; $\sqrt{50} = 5\sqrt{2}$; $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$

$\sqrt{\frac{9}{16}} = \frac{3}{4}$; $(\sqrt{3})^2 = 3$; $3 + \sqrt{12} = 3 + 2\sqrt{3}$

Answer: $\sqrt{0.25}$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $\sqrt{\frac{9}{16}}$, $(\sqrt{3})^2$

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.

(a) Work out the total length of the boundary of the track.

(b) Work out the area enclosed by the track. Give your answers in terms of π.

The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 10: circumference $= 2 \times \pi \times 10 = 20\pi$

The two straight sides: $2 \times 120 = 240$

Boundary = $(240 + 20\pi)$ m

(b) The rectangle is 120 m by $2 \times 10 = 20$ m, so its area is $120 \times 20 = 2400$. Circle: $\pi \times 10^2 = 100\pi$

Area = $(2400 + 100\pi)$ m^2

Answer: (a) $(240 + 20\pi)$ m (b) $(2400 + 100\pi)$ m^2

5 Weighted percentages

The ratio of juniors to seniors in a school is 2 : 1.

40% of the juniors and 10% of the seniors play a sport.

What percentage of all of these students play a sport?

Imagine 60 students in total: 40 juniors and 20 seniors

40% of 40 = 16 and 10% of 20 = 2

Together: 18 out of 60

$18 \div 60 \times 100 = 30\%$

Answer: 30%

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6 Surds

Given that $\sqrt{6} = \sqrt{n}$,
find the value of n.

$\sqrt{6} = \sqrt{1} \times \sqrt{6} = \sqrt{6}$
n = 6
Answer: n = 6

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (−4, 8).
The curve crosses the y-axis at (0, −24).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (−4, 8), so $y = a(x + 4)^2 + 8$
When x = 0, y = −24: a × 16 + 8 = −24, so a = −2
$y = -2(x + 4)^2 + 8$
Expand: $y = -2x^2 - 16x - 24$
Answer: $y = -2x^2 - 16x - 24$

8 Bounds of speed, density and pressure

A force of 410 N acts on a surface, correct to 2 significant figures.
The area of the surface is 5.7 m^2, correct to 2 significant figures.
Work out the lower bound of the pressure.
Give your answer correct to 3 significant figures. (Calculator)

405 ≤ force < 415; 5.65 ≤ area < 5.75
Lower bound of pressure = smallest force ÷ biggest area = 405 ÷ 5.75
= 70.4348... = 70.4 (3 s.f.)
Answer: 70.4 N/m^2

9 Algebraic proof (harder)

Prove algebraically that $k^3 + 23k$ is a multiple of 6 for all integer values of k.

$k^3 + 23k = (k^3 - k) + 24k = (k - 1)k(k + 1) + 24k$
(k − 1)k(k + 1) is the product of three consecutive integers, so it contains a multiple of 2 and a multiple of 3, so it is a multiple of 6
24k = 6 × 4k is also a multiple of 6
The sum of two multiples of 6 is a multiple of 6
Answer: Proved

10 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 41° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$
$\frac{\sin\theta}{6} = \frac{\sin 41°}{8}$
$\sin\theta = \frac{6 \times \sin 41°}{8} = 0.49204$
$\theta = \sin^{-1}(0.49204) = 29.5°$ (1 d.p.)
Answer: $\theta = 29.5°$