Day 148 **S Tier 10-a-day: Model Solutions**

1 Percentage change in area and volume

The side length of a square is decreased by 30%.
Work out the percentage change in the area of the square.

Side length multiplier = 0.7

Area multiplier = $0.7^2 = 0.49$

$0.49 - 1 = -0.51$, so a 51% decrease

Answer: 51% decrease

2 Factorising fully

Factorise fully $(x + 3)^2 - 25$

This is a difference of two squares: $(x + 3)^2 - 5^2$

$= (x + 3 - 5)(x + 3 + 5)$

$= (x - 2)(x + 8)$

Answer: $(x - 2)(x + 8)$

3 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 4 cm and the area of triangle ADE is 20 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 4 = 8 cm

Length scale factor = 8 ÷ 4 = 2

Area scale factor = $(2)^2 = 4$

Area of triangle ABC = $20 \times 4 = 80$ cm^2

Trapezium = $80 - 20 = 60$ cm^2

Answer: 60 cm^2

4 Standard form

Work out $(2.55 \times 10^{-7}) \div (1.5 \times 10^7)$
Give your answer in standard form.

$2.55 \div 1.5 = 1.7$

$10^{-7} \div 10^7 = 10^{-14}$

$= 1.7 \times 10^{-14}$

Answer: 1.7×10^{-14}

5 Conditional probability from a table

The table shows information about 115 children.
(a) A child who chooses tennis is chosen at random. Work out the probability that this child is a boy.
(b) Two of the 115 children are chosen at random. Work out the probability that both are boys.

(a) 49 children are in the Tennis column, and 26 of them are boys

Probability = $\frac{26}{49}$

(b) There are 55 boys, so P(both) = $\frac{55}{115} \times \frac{54}{114} = \frac{99}{437}$

Answer: (a) $\frac{26}{49}$ (b) $\frac{99}{437}$

Day 148 **S Tier 10-a-day: Model Solutions**

6 Trigonometric graphs

The diagram shows the graph of y = a sin bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 170°.

(a) The maximum value is 3, so a = 3
The graph completes 3 full cycles between 0° and 360°, so b = 3
(b) y = 3 sin (3 × 170°) = 3 sin 510°
sin 150° = $\frac{1}{2}$, so y = $\frac{3}{2}$

Answer: (a) a = 3, b = 3 (b) y = $\frac{3}{2}$

7 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 10, 4, 6, 32
Mid-points: 5, 20, 45, 80
Σfm = 10×5 + 4×20 + 6×45 + 32×80 = 2960, Σf = 52
Mean = 2960 ÷ 52 = 56.9
Answer: 56.9

8 Finding the rate of interest

Dina invested £18,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £19,626.65.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so 18500 × m^2 = 19626.65
m^2 = 19626.65 ÷ 18500 = 1.0609
m = √1.0609 = 1.03
Rate = (1.03 − 1) × 100 = 3%
Answer: 3% per year

9 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 15 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 5 cm from A
AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A

The bisector meets the arc where cos θ = $\frac{5}{10}$, so θ = 60° at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 5 cm, height 10 sin 60° = $5\sqrt{3}$ cm, area = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 10^2 = \frac{25}{3}\pi$

Area = $\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi$ cm^2

Answer: ($\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi$) cm^2

Day 148 **S Tier 10-a-day: Model Solutions**

10 Perpendicular bisector

A is the point $(-3, -4)$ and B is the point $(-5, 4)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Midpoint of AB = $\left(\frac{-3 + (-5)}{2}, \frac{-4 + 4}{2}\right) = (-4, 0)$

Gradient of AB = $\frac{4 - (-4)}{-5 - (-3)} = -4$

The perpendicular gradient is $-1 \div (-4) = \frac{1}{4}$

Using the midpoint: $y = \frac{1}{4}(x + 4)$

$y = \frac{1}{4}x + 1$

Answer: $y = \frac{1}{4}x + 1$