Day 147 **S Tier 10-a-day: Model Solutions**

1 Simplifying surds

Write $\sqrt{252} + \sqrt{175} + \sqrt{7}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{252} = 6\sqrt{7}$; $\sqrt{175} = 5\sqrt{7}$
$(6 + 5 + 1)\sqrt{7} = 12\sqrt{7}$
Answer: $12\sqrt{7}$

2 Combined transformations

The point P(3, −5) is rotated 90° anticlockwise about the origin.
The image is then reflected in the y-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (5, 3)
After the second transformation: P'' = (−5, 3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, x) → (y, x)
The single mapping is (x, y) → (y, x)
This is a reflection in the line y = x
Answer: (a) (−5, 3) (b) Reflection in the line y = x

3 Factorising fully

Factorise fully $5x^2 + 50x + 105$

Take out the common factor 5: $5(x^2 + 10x + 21)$
Factorise the quadratic: $5(x + 7)(x + 3)$
Answer: $5(x + 7)(x + 3)$

4 Area of a triangle using sine

Triangle ABC has AB = 9 cm and BC = 20 cm.
The area of triangle ABC is $45\sqrt{3}$ cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$45\sqrt{3} = \frac{1}{2} \times 9 \times 20 \times \sin B$

$\sin B = 45\sqrt{3} \div 90 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is acute, so B = 60°

Answer: 60°

5 Speed-time graphs

The graph shows the speed of a car during 20 seconds.
(a) Work out the acceleration during the first 8 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = (20 − 4) ÷ 8 = 2 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{4 + 20}{2} \times 8 = 96$; rectangle: $20 \times 8 = 160$; triangle: $\frac{1}{2} \times 4 \times 20 = 40$
Total = 296 m, so average speed = 296 ÷ 20 = 14.8 m/s
Answer: (a) 2 m/s^2 (b) 14.8 m/s

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6 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 4x + 3$

$y = -7x + 1$

Substitute: $x^2 - 4x + 3 = -7x + 1$

Rearrange: $x^2 + 3x + 2 = 0$

Factorise: $(x + 2)(x + 1) = 0$, so $x = -2$ or $x = -1$

When $x = -2$: $y = -7 \times (-2) + 1 = 15$

When $x = -1$: $y = -7 \times (-1) + 1 = 8$

Answer: $x = -2$, $y = 15$ and $x = -1$, $y = 8$

7 Histograms with an unmarked scale

The histogram shows the times of people spent waiting at a clinic.
120 of them had a time between 30 and 50 minutes.
(a) How many had a time between 10 and 20 minutes?
(b) How many people are there in total?

Frequency is proportional to the area of a bar
30 to 50: bar is 2 squares high and 20 wide, area 40, which represents 120
So 1 unit of area represents $120 \div 40 = 3$
(a) 10 to 20: $2 \times 10 = 20$ units of area, so $20 \times 3 = 60$
(b) Total area $= 70 + 20 + 50 + 40 = 180$, so total $= 180 \times 3 = 540$
Answer: (a) 60 (b) 540

8 Speed and a moving train

A train travels at a constant speed of 72 km/h.
It takes 20 seconds to go completely through a tunnel which is 130 m long.
Work out the length of the train, in metres.

Speed = 72 km/h = $72{,}000 \div 3600 = 20$ m/s
Distance travelled in 20 s $= 20 \times 20 = 400$ m
This is the length of the tunnel plus the length of the train
Train $= 400 - 130 = 270$ m
Answer: 270 m

9 Counting factors

$N = 2 \times 3^5$

How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 1, that is 1 choices
Power of 3: 0 to 5, that is 6 choices
Total $= 1 \times 6 = 6$
Answer: 6

10 Perpendicular bisector

A is the point (4, 5) and B is the point (−4, 9).
Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = (0, 7)

Gradient of AB = $-\frac{1}{2}$

Perpendicular gradient = 2
$y - 7 = 2(x - 0)$
Rearrange to integer coefficients: $2x - y = -7$
Answer: $2x - y = -7$