Day 146 **S Tier 10-a-day: Model Solutions**

1 Radius from chord and height of arc

A circular arc has a chord AB of length 48 cm.
The greatest distance between the chord and the arc is 8 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 8$

AM = $48 \div 2 = 24$. Pythagoras in triangle OMA: $r^2 = (r - 8)^2 + 24^2$

$r^2 = r^2 - 16r + 64 + 576$

$16r = 640$, so $r = 40$ cm

Answer: 40 cm

2 Tests of divisibility

The 4-digit number 1k53 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9

Digit sum = $1 + 5 + 3 + k = 9 + k$

This must be a multiple of 9, so k = 0 or 9

Answer: k = 0 or 9

3 Three-part ratios

Sofia, Priya and Oscar share some money.
Sofia : Priya = 5 : 6
Priya : Oscar = 2 : 9
(a) Work out the ratio Sofia : Priya : Oscar.
(b) They share £190 altogether. How much money does Priya get?

(a) Make the Priya part the same: 5 : 6 = 5 : 6 and 2 : 9 = 6 : 27

Sofia : Priya : Oscar = 5 : 6 : 27

(b) Total parts = 38, so 1 part = $190 \div 38 = 5$

Priya gets $6 \times 5 = £30$

Answer: (a) 5 : 6 : 27 (b) £30

4 Equations with indices

Solve $32^{2x+3} = 2^{2x-1}$

Write both sides as powers of 2: $(2^5)^{2x+3} = (2^1)^{2x-1}$

$2^{10x+15} = 2^{2x-1}$

Equate the powers: $10x + 15 = 2x - 1$

$8x = -16$

$x = -2$

Answer: x = −2

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 6 cm, DB = 8 cm and the area of triangle ADE is 81 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 6 + 8 = 14 cm

Length scale factor = $14 \div 6 = \frac{7}{3}$

Area scale factor = $(\frac{7}{3})^2 = \frac{49}{9}$

Area of triangle ABC = $81 \times \frac{49}{9} = 441$ cm^2

Trapezium = $441 - 81 = 360$ cm^2

Answer: 360 cm^2

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6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 8 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 24 ÷ 8 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 8 \times 24 + 24 \times 20 + \frac{1}{2} \times 6 \times 24 = 96 + 480 + 72$
= 648
Answer: (a) 3 m/s^2 (b) 648 m

7 Density of an alloy

An alloy contains 213 g of zinc and 54 g of aluminium.
The density of zinc is 7.1 g/cm^3. The density of aluminium is 2.7 g/cm^3.
Work out the density of the alloy.

Volume of zinc = 213 ÷ 7.1 = 30 cm^3
Volume of aluminium = 54 ÷ 2.7 = 20 cm^3
Total mass = 267 g, total volume = 50 cm^3
Density = 267 ÷ 50 = 5.34 g/cm^3
Answer: 5.34 g/cm^3

8 Bounds of roots

The area of a square is 231 cm^2, correct to the nearest cm^2.
Work out the lower bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

230.5 ≤ area < 231.5
Lower bound of side = √230.5
= 15.1822... = 15.2 (3 s.f.)
Answer: 15.2 cm

9 Exact trigonometric ratios from a given ratio

x is an acute angle and tan x = $\frac{7}{10}$.

(a) Find the exact value of sin x.
(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 7 and adjacent side 10
Hypotenuse = $\sqrt{7^2 + 10^2} = \sqrt{149}$
(a) sin x = $\frac{7}{\sqrt{149}} = \frac{7\sqrt{149}}{149}$
(b) cos x = $\frac{10}{\sqrt{149}} = \frac{10\sqrt{149}}{149}$
Answer: (a) $\frac{7\sqrt{149}}{149}$ (b) $\frac{10\sqrt{149}}{149}$

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10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.6.
If it rains, the probability that Ali is late for school is 0.4.
If it does not rain, the probability that Ali is late is 0.3.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.6 × 0.4 + 0.4 × 0.3 = 0.24 + 0.12 = 0.36
(b) P(rain and late) = 0.24

P(rain | late) = 0.24 ÷ 0.36 = $\frac{2}{3}$

Answer: (a) 0.36 (b) $\frac{2}{3}$