Day 145 **S Tier 10-a-day: Model Solutions**

1 Laws of indices

Given that $\frac{25^3 \times 5^4}{5^3} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^6 \times 5^4 = 5^{10}$

Denominator: 5^3

Subtract the powers: $10 - 3 = 7$

Answer: n = 7

2 Factorising quadratics

Factorise $4x^2 + 8x - 5$

$4 \times (-5) = -20$. Two numbers that multiply to -20 and add to 8 are -2 and 10

Split the middle term: $4x^2 - 2x + 10x - 5$

Factorise in pairs: $2x(2x - 1) + 5(2x - 1)$

$= (2x + 5)(2x - 1)$

Answer: (2x + 5)(2x − 1)

3 Percentages

R is 30% more than Q.

Q is 25% less than P.

Work out R as a percentage of P.

$R = 1.3 \times Q$ and $Q = 0.75 \times P$

$R = 1.3 \times 0.75 \times P = 0.975 \times P$

$0.975 \times 100 = 97.5\%$

Answer: 97.5%

4 Averages with algebra

Here are five numbers: $x - 3$, $x - 2$, $x + 2$, 2, $3x - 2$.

The mean of the five numbers is 9.

(a) Work out the value of x.

(b) Work out the range of the five numbers.

(a) Total = $(x - 3) + (x - 2) + (x + 2) + (2) + (3x - 2) = 9 \times 5 = 45$

$6x - 3 = 45$, so $6x = 48$ and $x = 8$

The numbers are 5, 6, 10, 2, 22

(b) Range = $22 - 2 = 20$

Answer: (a) x = 8 (b) 20

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.

1, −4, −11, −20, −31

Find an expression for the nth term.

First differences: −5, −7, −9, −11

Second differences: −2, −2, −2, so $a = -2 \div 2 = -1$

Subtract $-1n^2$ from each term: 2, 0, −2, −4, −6

The remaining linear sequence has nth term $-2n + 4$

nth term = $-n^2 - 2n + 4$

Answer: $-n^2 - 2n + 4$

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6 HCF and LCM

The HCF of two numbers is 7 and their LCM is 252.
One of the numbers is 28.
Work out the other number.

HCF × LCM = product of the two numbers
7 × 252 = 1764
Other number = 1764 ÷ 28 = 63
Answer: 63

7 Area and quadratic equations

The diagram shows a rectangle of length (2x + 6) cm, width (2x + 5) cm and area 132 cm^2.
(a) Show that $4x^2 + 22x - 102 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) (2x + 6)(2x + 5) = 132
Expand: $4x^2 + 22x + 30 = 132$, so $4x^2 + 22x - 102 = 0$
(b) Factorise: 2(x − 3)(2x + 17) = 0
x = 3 ($x = -\frac{17}{2}$ is not possible as a length would be negative)
Length = 12 cm and width = 11 cm
Diagonal = $\sqrt{12^2 + 11^2} = \sqrt{265}$ = 16.3 cm (3 s.f.)
Answer: (b) 16.3 cm

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 7x + 12}{x - 1} \div \frac{x + 3}{x^2 + 4x - 5}$

Factorise: $\frac{(x + 4)(x + 3)}{(x - 1)} \div \frac{(x + 3)}{(x - 1)(x + 5)}$

Turn the second fraction upside down and multiply: $\frac{(x + 4)(x + 3)}{(x - 1)} \times \frac{(x - 1)(x + 5)}{(x + 3)}$

Cancel (x + 3) and (x − 1)
= (x + 4)(x + 5)
Answer: (x + 4)(x + 5)

9 Ratio and money transferred

Oscar and Liam share some money in the ratio 8 : 5.
Oscar gives Liam £4.
The ratio of their money is now 3 : 2.
How much money do they have altogether?

Let the amounts be 8k and 5k, so the total is 13k
Oscar now has 8k − 4, which is $\frac{3}{5}$ of the total
5(8k − 4) = 3 × 13k
40k − 20 = 39k, so 1k = 20 and k = 20
Total = 13 × 20 = £260
Answer: £260

10 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 45°
tan 45° = 1
$x = 12\sqrt{2} \times \tan 45° = 12\sqrt{2} \times 1$
$x = 12\sqrt{2}$
Answer: $x = 12\sqrt{2}$