Day 144 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 07:05 (London time).
It takes 6 hours 40 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 07:05 + 6 h 40 min = 13:45
Sydney is 9 hours ahead of London, so add 9 hours
22:45
Answer: 22:45 the same day

2 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (2, 36) and (4, 144).
Find the value of k and the value of a.

Substitute the points: $k\,a^2 = 36$ and $k\,a^4 = 144$

Divide: $a^2 = \frac{144}{36} = 4$

$a = 2$ (positive, since $a > 0$)
From the first point: $k \times 4 = 36$, so $k = 9$
Answer: $k = 9$, $a = 2$

3 Standard form

Work out $(2.4 \times 10^{-3}) - (4.3 \times 10^{-6})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-6}: $2.4 \times 10^{-3} = 2400 \times 10^{-6}$
$(2400 - 4.3) \times 10^{-6} = 2395.7 \times 10^{-6}$
$= 2.3957 \times 10^{-3}$
Answer: 2.3957×10^{-3}

4 Relative speed

Isla sets off at 9:00 and cycles at 8 km/h along a path.
Hassan sets off from the same place at 9:45 and cycles at 16 km/h along the same path.
At what time does Hassan catch up with Isla?

When Hassan starts, Isla is $0.75 \times 8 = 6$ km ahead
Hassan gains $16 - 8 = 8$ km every hour
Time to catch up = $6 \div 8 = 0.75$ hours = 45 minutes
9:45 + 45 minutes = 10:30
Answer: 10:30

5 Equations with indices

Solve $x^{-\frac{1}{2}} = \frac{1}{3}$

Write $\frac{1}{3}$ as a power of 3: $\frac{1}{3} = 3^{-1}$

$x^{-\frac{1}{2}} = 3^{-1}$, so $x = (3^{-1})^{-2}$
$x = 3^2 = 9$
Answer: $x = 9$

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6 Area of a segment

The circle has centre O and radius 7 cm. A and B are points on the circle and angle AOB = 70°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{70}{360} \times \pi \times 7^2 = 29.932$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 7 \times 7 \times \sin 70° = 23.022$ cm^2

Segment = sector − triangle = 29.932 − 23.022 = 6.9099

= 6.91 cm^2 (3 s.f.)

Answer: 6.91 cm^2

7 Density of an alloy

An alloy contains 73 g of tin and 108 g of aluminium.
The density of tin is 7.3 g/cm^3. The density of aluminium is 2.7 g/cm^3.
Work out the density of the alloy.

Volume of tin = 73 ÷ 7.3 = 10 cm^3

Volume of aluminium = 108 ÷ 2.7 = 40 cm^3

Total mass = 181 g, total volume = 50 cm^3

Density = 181 ÷ 50 = 3.62 g/cm^3

Answer: 3.62 g/cm^3

8 Exact lengths and areas

A rectangle has area $(18 + 8\sqrt{5})$ cm^2 and length $(2 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{18 + 8\sqrt{5}}{2 + \sqrt{5}}$

Multiply top and bottom by $(2 - \sqrt{5})$:

Numerator: $(18 + 8\sqrt{5})(2 - \sqrt{5}) = -4 - 2\sqrt{5}$

Denominator: $4 - 5 = -1$

Width = $4 + 2\sqrt{5}$

Answer: $4 + 2\sqrt{5}$ cm

9 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 7.
(b) Given that at least one number is 3, work out the probability that the sum is greater than 7.

(a) There are 4 × 6 = 24 equally likely outcomes, and 6 of them satisfy: the sum is greater than 7

Probability = $\frac{1}{4}$

(b) 9 outcomes satisfy: at least one number is 3. Of these, 2 also satisfy: the sum is greater than 7

Probability = $\frac{2}{9}$

Answer: (a) $\frac{1}{4}$ (b) $\frac{2}{9}$

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10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $3\mathbf{a} - 3\mathbf{b}$, OQ = $\mathbf{a} + 3\mathbf{b}$ and OR = $6\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-2\mathbf{a} + 6\mathbf{b}$
QR = OR − OQ = $-\mathbf{a} + 3\mathbf{b}$

QR = $\frac{1}{2}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 1
Answer: (b) 2 : 1