Day 143 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-2}{6} + \frac{x-1}{8} = 1$

Multiply every term by 24: $4(x-2) + 3(x-1) = 24$
Expand: $7x - 11 = 24$
$7x = 35$
$x = 5$
Answer: $x = 5$

2 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = 0, OQ = $12\mathbf{a} - 9\mathbf{b}$ and OR = $28\mathbf{a} - 21\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $12\mathbf{a} - 9\mathbf{b}$
QR = OR − OQ = $16\mathbf{a} - 12\mathbf{b}$

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4
Answer: (b) 3 : 4

3 Standard form problems

The mass of one coin is 1.25×10^{-2} g.
A box of coins has a total mass of 7.875×10^{5} g.
Work out how many there are in the box.
Give your answer in standard form.

Number = total mass ÷ mass of one
$(7.875 \times 10^{5}) \div (1.25 \times 10^{-2})$
$7.875 \div 1.25 = 6.3$ and $10^{5} \div 10^{-2} = 10^{7}$
$= 6.3 \times 10^{7}$
Answer: 6.3×10^{7}

4 Trigonometric graphs

The diagram shows the graph of y = a sin x + c.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when x = 210°.

(a) The maximum value is 6 and the minimum value is 0
a = (6 − (0)) ÷ 2 = 3
c = (6 + (0)) ÷ 2 = 3

(b) y = 3 sin 210° + 3, and sin 210° = $-\frac{1}{2}$

y = $\frac{3}{2}$

Answer: (a) a = 3, c = 3 (b) y = $\frac{3}{2}$

5 Transformations of graphs

The point P(−6, −4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(2x) (b) y = 2f(x)

(a) y = f(2x) is a horizontal stretch with scale factor $\frac{1}{2}$, so the x coordinate is halved: (−3, −4)

(b) y = 2f(x) is a vertical stretch with scale factor 2, so the y coordinate is doubled: (−6, −8)
Answer: (a) (−3, −4) (b) (−6, −8)

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6 Distance-time graphs

The graph shows Fatima's cycle ride to a café and back.
(a) Work out Fatima's speed on the way back.
(b) Work out Fatima's average speed for the whole ride, including the stop.

(a) Return: 10 km in 0.5 hours, so speed = 10 ÷ 0.5 = 20 km/h
(b) Total distance = 2 × 10 = 20 km, total time = 2 hours
Average speed = 20 ÷ 2 = 10 km/h
Answer: (a) 20 km/h (b) 10 km/h

7 Upper and lower bounds

a = 9.5 and b = 4.1,
both correct to 1 decimal place.
Work out the lower bound of a + b.

Each value is rounded, so it can be up to 0.05 above or below
$9.45 \leq a < 9.55$; $4.05 \leq b < 4.15$
Lower bound = 9.45 + 4.05 = 13.5
Answer: 13.5

8 Pythagoras with algebra

ABC is a right-angled triangle with sides (x) cm, (2x − 4) cm and (x + 4) cm. The longest side is (x + 4) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $x^2 + (2x - 4)^2 = (x + 4)^2$
Expand and collect terms: $4x^2 - 24x = 0$
Factorise: $4x(x - 6) = 0$
x = 6 or x = 0, and x must be positive, so x = 6
(b) The two shorter sides are 6 cm and 8 cm
Area = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2
Answer: (a) x = 6 (b) 24 cm^2

9 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 9.75%.
Work out the value of x.

Yearly multiplier = $1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.9025$

Take the square root: $1 - \frac{x}{100} = 0.95$

$\frac{x}{100} = 0.05$, so x = 5
Answer: x = 5

10 Percentage change and proportion

The volume of a cube is directly proportional to the cube of the length of its edge.
The length of its edge is increased by 50%.
Work out the percentage change in the volume of a cube.

The length of its edge is multiplied by 1.5
The multiplier for the volume of a cube is $1.5^3 = 3.375$
3.375 is an increase of 237.5%
Answer: 237.5% increase