Day 142 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 23°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 23°
Angle AOB = 180 − 2 × 23 = 134°
(b) The angle at the circumference is half the angle at the centre: 134 ÷ 2 = 67°
Answer: (a) 134° (b) 67°

2 Transformations of graphs

The point P(8, 4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x) + 4 (b) y = f(−x)

(a) y = f(x) + 4 is a translation of 4 up: (8, 8)
(b) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (−8, 4)
Answer: (a) (8, 8) (b) (−8, 4)

3 Rationalising the denominator

Rationalise the denominator of $\frac{29}{2\sqrt{6}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:
$\frac{29}{2\sqrt{6}} = \frac{29\sqrt{6}}{12}$
Answer: $\frac{29\sqrt{6}}{12}$

4 Exchange rates

Tariq changes £400 into dollars at £1 = $1.15.
Tariq spends $100 and then changes the rest back into pounds at £1 = $1.6.
How many pounds does Tariq get back?

£400 × 1.15 = $460
460 − 100 = $360
360 ÷ 1.6 = £225
Answer: £225

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 8 cm, DB = 12 cm and the area of triangle ADE is 96 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 8 + 12 = 20 cm
Length scale factor = 20 ÷ 8 = $\frac{5}{2}$
Area scale factor = $(\frac{5}{2})^2 = \frac{25}{4}$
Area of triangle ABC = 96 × $\frac{25}{4}$ = 600 cm^2
Trapezium = 600 − 96 = 504 cm^2
Answer: 504 cm^2

Day 142 **S Tier 10-a-day: Model Solutions**

6 Percentage change in area and volume

The length of a rectangle is decreased by 30%.
The width of the rectangle is increased by 30%.
Work out the percentage change in the area of the rectangle.

Length multiplier 0.7, width multiplier 1.3
Area multiplier = 0.7 × 1.3 = 0.91
0.91 − 1 = −0.09, so a 9% decrease
Answer: 9% decrease

7 Winning a match

Maya and Amir play a match. The first to win 2 chess games wins the match.
The probability that Maya wins any one of the chess games is $\frac{2}{5}$.
Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two
P(Maya wins then loses) = $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$
P(Maya loses then wins) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$
Total = $2 \times \frac{6}{25} = \frac{12}{25}$
Answer: $\mathbf{\frac{12}{25}}$

8 Index equations

Solve $27^x = 3^{2x-1}$

Write both sides as powers of 3: $27 = 3^3$
Equate the powers: $3x = 2x - 1$
$x = -1$
Answer: $x = -1$

9 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 60 seconds to go completely through a tunnel which is 1,120 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 60 s = 25 × 60 = 1,500 m
This is the length of the tunnel plus the length of the train
Train = 1,500 − 1,120 = 380 m
Answer: 380 m

10 Equations with algebraic fractions

Solve $\frac{3}{x+2} - \frac{3}{x+5} = \frac{1}{2}$

Multiply every term by $(x + 2)(x + 5)$: $3(x + 5) - 3(x + 2) = \frac{1}{2} \times (x + 2)(x + 5)$
Multiply by 2 to clear the fraction: $6(x + 5) - 6(x + 2) = (x + 2)(x + 5)$
Expand and rearrange so that one side is 0: $x^2 + 7x - 8 = 0$
Factorise: $(x - 1)(x + 8) = 0$
$x = -8$ or $x = 1$
Answer: $x = -8$ or $x = 1$