Day 141 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 174°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 174° = 6°
Number of sides = 360° ÷ 6° = 60
Answer: 60

2 Prime factors and indices

Write $8^5 \times 25^4$ as a product of powers of prime numbers.

$8 = 2^3$, $25 = 5^2$
$8^5 = 2^{15}$
$25^4 = 5^8$
Add the powers of each prime: $2^{15} \times 5^8$
Answer: $2^{15} \times 5^8$

3 Composite functions: fg and gf

$f(x) = -3x + 1$ and $g(x) = x^2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2) = -3x^2 + 1$
$gf(x) = g(-3x + 1) = (-3x + 1)^2 = 9x^2 - 6x + 1$
Set them equal and rearrange: $12x^2 - 6x = 0$
Divide by 6 and factorise: $x(2x - 1) = 0$
$x = 0$ or $x = \frac{1}{2}$
Answer: $x = 0$ or $x = \frac{1}{2}$

4 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 37° and angle CAQ = 40°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 40° (alternate segment theorem)
(b) Angle ACB = angle PAB = 37° (alternate segment theorem)
Answer: (a) 40° (b) 37°

5 Histograms with an unmarked scale

The histogram shows the lengths of leaves collected from a tree.
250 of them had a length between 0 and 10 cm.
(a) How many had a length between 10 and 20 cm?
(b) How many leaves are there in total?

Frequency is proportional to the area of a bar
0 to 10: bar is 5 squares high and 10 wide, area 50, which represents 250
So 1 unit of area represents 250 ÷ 50 = 5
(a) 10 to 20: 5 × 10 = 50 units of area, so 50 × 5 = 250
(b) Total area = 50 + 50 + 40 + 60 = 200, so total = 200 × 5 = 1000
Answer: (a) 250 (b) 1000

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6 Exact lengths and areas

A square has sides of length $(2 + \sqrt{2})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{2}$

Area $= (2 + \sqrt{2})^2$
$= 4 + 4\sqrt{2} + 2$
$= 6 + 4\sqrt{2}$
Answer: $6 + 4\sqrt{2}$ cm^2

7 Direct and inverse proportion

y is inversely proportional to x.
When x = 8, y = 1.
(a) Find a formula for y in terms of x.
(b) Find the value of x when y = 4.

(a) $y = \frac{k}{x}$

Substitute x = 8 and y = 1: $1 = \frac{k}{8}$
$k = 8$
$y = \frac{8}{x}$

(b) $4 = \frac{8}{x}$, so $x = 8 \div 4 = 2$

Answer: (a) $y = \frac{8}{x}$ (b) x = 2

8 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 24 cm, top radius 2 cm and base radius 8 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h + 24} = \frac{2}{8}$
$8h = 2(h + 24)$, so $6h = 48$ and $h = 8$
Large cone: $\frac{1}{3} \times \pi \times 8^2 \times 32 = \frac{2048}{3}\pi$
Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 8 = \frac{32}{3}\pi$
Frustum $= \frac{2048}{3}\pi - \frac{32}{3}\pi = 672\pi$ cm^3
Answer: 672π cm^3

9 Nested surds

Show that $\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$

Square the right-hand side: $(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$
So $\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$
Answer: $\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$

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10 Probability from a frequency tree

In a population of 20,000 people, 5% have a certain disease.
A test gives a positive result for 95% of people who have the disease and for 10% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

5% of 20,000 = 1,000 have the disease and 19,000 do not
Flagged and genuine: 1,000 × 95% = 950
Flagged but not: 19,000 × 10% = 1,900
Total flagged or positive = 950 + 1,900 = 2,850

Probability = 950 ÷ 2,850 = $\frac{1}{3}$

Answer: $\mathbf{\frac{1}{3}}$