Day 140 **S Tier 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of $\sin 45° \times \tan 30°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\sin 45° \times \tan 30° = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}$

$= \frac{\sqrt{6}}{6}$

Answer: $\frac{\sqrt{6}}{6}$

2 Ordering numbers

Write these numbers in order of size, starting with the largest.
38%, $\frac{2}{5}$, 0.61, 51%

Convert each to a decimal: 38% = 0.38, $\frac{2}{5}$ = 0.4, 0.61 = 0.61, 51% = 0.51

In order: 0.61, 51%, $\frac{2}{5}$, 38%

Answer: 0.61, 51%, $\frac{2}{5}$, 38%

3 Gradient of a chord

A and B are points on the curve $y = x^3 - x$.
The x coordinate of A is -3 and the x coordinate of B is 0.
Work out the gradient of the line AB.

At A: $y = -24$. At B: $y = 0$
A is $(-3, -24)$ and B is $(0, 0)$
Gradient = $\frac{0 - (-24)}{0 - (-3)} = \frac{24}{3} = 8$

Answer: 8

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(4x + 15)°$, angle BCD = $(3x - 17)°$ and angle ABC = $(2x + 10)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(4x + 15) + (3x - 17) = 180$
$7x - 2 = 180$, so $7x = 182$
$x = 26$
(b) Angle ABC = $2 \times 26 + 10 = 62°$
Angle ADC = $180 - 62 = 118°$
Answer: (a) x = 26 (b) 118°

5 Rational and irrational numbers

Here are six numbers.
$\sqrt{49}$, $(\sqrt{3})^2$, $\sqrt[3]{27}$, $\sqrt{50}$, $\sqrt{0.25}$, $\sqrt{3} \times \sqrt{5}$
Write down the irrational numbers.

Simplify each number:
$\sqrt{49} = 7$; $(\sqrt{3})^2 = 3$; $\sqrt[3]{27} = 3$
$\sqrt{50} = 5\sqrt{2}$; $\sqrt{0.25} = 0.5$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$
Answer: $\sqrt{50}$, $\sqrt{3} \times \sqrt{5}$

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6 Transformations of graphs

The graph of y = f(x) has a maximum point at (5, −4).
Write down the coordinates of the turning point of the graph of
(a) y = −f(x)
(b) y = 2f(−x)

(a) Start with the turning point (5, −4)
Apply the transformation to the point: (5, 4)
(b) f(−x) changes the x-coordinate to −5, and 2f(x) changes the y-coordinate to −8
Turning point: (−5, −8)
Answer: (a) (5, 4) (b) (−5, −8)

7 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-7-0}{0-(-3)} = -\frac{7}{3}$

Perpendicular gradient = $-1 \div (-\frac{7}{3}) = \frac{3}{7}$

M passes through B(−3, 0): $0 = \frac{3}{7} \times (-3) + c$, so $c = \frac{9}{7}$

$y = \frac{3}{7}x + \frac{9}{7}$

Answer: $y = \frac{3}{7}x + \frac{9}{7}$

8 Inverse proportion

y is inversely proportional to x.
When x = 5, y = 6.
Find the positive value of x when y = 15.

$y = \frac{k}{x}$

$6 = \frac{k}{5}$, so $k = 6 \times 5 = 30$

$15 = \frac{30}{x}$, so $x = \frac{30}{15} = 2$

Answer: x = 2

9 Functions that are their own inverse

$f(x) = \frac{x+3}{3x-1}$ where $x \neq \frac{1}{3}$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (3x − 1)

$ff(x) = \frac{1(x+3)+3(3x-1)}{3(x+3)-1(3x-1)}$

Top = x + 3 + 9x − 3 = 10x
Bottom = 3x + 9 − 3x + 1 = 10

$ff(x) = \frac{10x}{10} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{x+3}{3x-1}$

Answer: (b) $f^{-1}(x) = \frac{x+3}{3x-1}$

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10 Estimating the median from grouped data

The table gives information about 98 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 15, 37, 67, 80, 98

The median is the value at position 98 ÷ 2 = 49

It is in the class 20 to 30, with 37 values before it, so we need 12 more of the 30 values in this class

Median $\approx 20 + \frac{12}{30} \times 10 = 24$

Answer: 24