Day 14 **S Tier 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 25^2 - 24^2 = 49$
AM = 7 cm
AB = 2 × 7 = 14 cm

(b) sin(angle AOM) = $\frac{7}{25}$

Angle AOM = 16.26°, so angle AOB = 2 × 16.26 = 32.5°

Answer: (a) 14 cm (b) 32.5°

2 Equations with algebraic fractions

Solve $\frac{6}{x+4} + \frac{5}{x+1} = \frac{2}{3}$

Multiply every term by (x + 4)(x + 1): $6(x + 1) + 5(x + 4) = \frac{2}{3} \times (x + 4)(x + 1)$

Multiply by 3 to clear the fraction: $18(x + 1) + 15(x + 4) = 2(x + 4)(x + 1)$

Expand and rearrange so that one side is 0: $2x^2 - 23x - 70 = 0$

Factorise: $(x - 14)(2x + 5) = 0$

$x = -\frac{5}{2}$ or x = 14

Answer: $x = -\frac{5}{2}$ or x = 14

3 Exact trigonometric values

Work out the exact value of $(\tan 45°)^2 + \tan 30°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\tan 45°)^2 + \tan 30° = (1)^2 + \frac{\sqrt{3}}{3}$

$= 1 + \frac{\sqrt{3}}{3}$

Answer: $1 + \frac{\sqrt{3}}{3}$

4 Standard form

Work out $(2.8 \times 10^{-2}) - (6.5 \times 10^{-4})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-4}: $2.8 \times 10^{-2} = 280 \times 10^{-4}$

$(280 - 6.5) \times 10^{-4} = 273.5 \times 10^{-4}$

$= 2.735 \times 10^{-2}$

Answer: 2.735×10^{-2}

5 Gradient of a chord

A and B are points on the curve $y = x^3 - x$.
The x coordinate of A is 2 and the x coordinate of B is 4.
Work out the gradient of the line AB.

At A: y = 6. At B: y = 60
A is (2, 6) and B is (4, 60)

Gradient = $\frac{60 - 6}{4 - 2} = \frac{54}{2} = 27$

Answer: 27

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6 Tree diagrams without replacement

A bag contains 2 orange beads and 5 purple beads.
Two beads are taken at random without replacement.
Work out the probability that (a) they are different colours, (b) at least one is orange.

(a) P(different) = 1 − P(same) = $1 - \frac{11}{21} = \frac{10}{21}$

(b) P(at least one orange) = 1 − P(none) = $1 - \frac{5}{7} \times \frac{4}{6} = \frac{11}{21}$

Answer: (a) $\frac{10}{21}$ (b) $\frac{11}{21}$

7 Surds

Given that $\sqrt{175} - \sqrt{28} = \sqrt{n}$,
find the value of n.

$\sqrt{175} = 5\sqrt{7}$ and $\sqrt{28} = 2\sqrt{7}$
$5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7}$
$3\sqrt{7} = \sqrt{63}$, so n = 63
Answer: n = 63

8 Simultaneous equations

Solve the simultaneous equations
$-x - 4y = 21$
$-3x + 4y = -33$

To match the x terms: (1) × 3 gives $-3x - 12y = 63$
Subtract the equations to eliminate x: $-16y = 96$
$y = -6$
Substitute $y = -6$ into (1): $-x + 24 = 21$
$-x = -3$, so x = 3
Answer: x = 3, y = −6

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 756 × k is a square number.

$756 = 2^2 \times 3^3 \times 7$
For a square number every power must be a multiple of 2
Multiply by 3 × 7 to get $2^2 \times 3^4 \times 7^2$
k = 21
Answer: k = 21

10 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is a multiple of 3.
(b) Given that the sum is odd, work out the probability that the sum is a multiple of 3.

(a) There are 4 × 5 = 20 equally likely outcomes, and 7 of them satisfy: the sum is a multiple of 3
Probability = $\frac{7}{20}$

(b) 10 outcomes satisfy: the sum is odd. Of these, 3 also satisfy: the sum is a multiple of 3
Probability = $\frac{3}{10}$

Answer: (a) $\frac{7}{20}$ (b) $\frac{3}{10}$