Day 139 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 165°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 165° = 15°
Number of sides = 360° ÷ 15° = 24
Answer: 24

2 Capture and recapture

100 deer in a forest are caught, marked and put back.
Later, a second sample of 20 deer is caught. 5 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 20
This should match the proportion marked in the whole population: $\frac{5}{20} = \frac{100}{N}$
N = 100 × 20 ÷ 5 = 400
(b) No deer are born, die or leave between the samples, and the deer mix evenly
Answer: (a) 400 (b) The population stays the same between samples

3 Comparing costs

Zip Taxis has a fixed charge of £3.50 plus £5.50 per mile.
Go Cabs has a fixed charge of £35 plus £1 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 14 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when 3.50 + 5.50n = 35 + n
4.50n = 31.50, so n = 7
(b) Zip Taxis: £3.50 + £5.50 × 14 = £80.50
Go Cabs: £35 + £1 × 14 = £49
Go Cabs is cheaper by £31.50
Answer: (a) 7 (b) Go Cabs, by £31.50

4 Expanding brackets with surds

Expand and simplify $(5 + \sqrt{5})(4 + 2\sqrt{5})$
Give your answer in the form $p + q\sqrt{5}$

$(5 + \sqrt{5})(4 + 2\sqrt{5}) = 20 + 10\sqrt{5} + 4\sqrt{5} + 10$
$= 30 + 14\sqrt{5}$
Answer: $30 + 14\sqrt{5}$

5 Inverse proportion in context

24 machines can make a batch of bolts in 24 hours.
After 4 hours, 8 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 24 × 24 = 576 machine-hours
Work done = 24 × 4 = 96, so work left = 480
Workers left = 24 − 8 = 16
Time = 480 ÷ 16 = 30 hours
Answer: 30 hours

6 Percentage change in area and volume

The radius of a sphere is increased by 10%.
Work out the percentage change in the volume of the sphere.

Radius multiplier = 1.1
Volume multiplier = $1.1^3 = 1.331$
1.331 − 1 = 0.331, so a 33.1% increase
Answer: 33.1% increase

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7 Density of an alloy

An alloy contains 316 g of iron and 108 g of aluminium.
The density of iron is 7.9 g/cm^3. The density of aluminium is 2.7 g/cm^3.
Work out the density of the alloy.

Volume of iron = 316 ÷ 7.9 = 40 cm^3
Volume of aluminium = 108 ÷ 2.7 = 40 cm^3
Total mass = 424 g, total volume = 80 cm^3
Density = 424 ÷ 80 = 5.3 g/cm^3
Answer: 5.3 g/cm^3

8 Equal roots and real roots

Show that the equation $x^2 + 2kx - 5 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (2k)^2 - 4 \times 1 \times (-5)$
$= 4k^2 + 20$
$4k^2$ is never negative, so $4k^2 + 20$ is always at least 20, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

9 Solving trigonometric equations

Solve $2 \sin x = \sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\sin x = \frac{\sqrt{3}}{2}$

The related acute angle satisfies $\sin \theta = \frac{\sqrt{3}}{2}$, so $\theta = 60°$
sine is positive in the 1st and 2nd quadrants, so $x = 60°$ and $x = 180° - 60°$
$x = 60°$ and $x = 120°$
Answer: x = 60° or x = 120°

10 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £20,000 is taxed at 20%.
All income above that is taxed at 40%.
Nia pays £18,600 in income tax in a year.
Work out Nia's income for the year.

Tax on the 20% band = 20% of £20,000 = £4,000
Tax paid at 40% = £18,600 − £4,000 = £14,600
Income taxed at 40% = £14,600 ÷ 0.4 = £36,500
Income = £15,000 + £20,000 + £36,500 = £71,500
Answer: £71,500