Day 138 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 42 is taken. Work out how many come from Group D.
(b) A different stratified sample has 14 from Group C. Work out the size of this sample.

Total = 300 + 300 + 175 + 275 = 1050

(a) Group D: $\frac{275}{1050} \times 42 = 11$

(b) Fraction sampled = $14 \div 175 = \frac{2}{25}$, so sample size = $\frac{2}{25} \times 1050 = 84$

Answer: (a) 11 (b) 84

2 Equations with fractions

Solve $\frac{x+7}{3} + \frac{x+4}{2} = 6$

Multiply every term by 6: $2(x + 7) + 3(x + 4) = 36$
Expand: $5x + 26 = 36$
$5x = 10$
$x = 2$
Answer: x = 2

3 Similar shapes

Two shapes are mathematically similar.
The smaller has height 18 cm and area 12 cm^2.
The larger has area $\frac{100}{3}$ cm^2.
Work out the height of the larger shape.

Ratio of areas = $\frac{100}{3} : 12 = 25 : 9$

Ratio of lengths = 5 : 3

Height = $18 \times \frac{5}{3} = 30$ cm

Answer: 30 cm

4 Rational and irrational numbers

Here are six numbers.
$0.\dot{4}$, $\sqrt{49}$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$, $(\sqrt{3})^2$, $3 + \sqrt{12}$, $\sqrt{2} \times \sqrt{6}$
Write down the irrational numbers.

Simplify each number:

$0.\dot{4} = \frac{4}{9}$; $\sqrt{49} = 7$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$

$(\sqrt{3})^2 = 3$; $3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$

Answer: $3 + \sqrt{12}$, $\sqrt{2} \times \sqrt{6}$

5 Enlargement

Triangle P has vertices A(5, 5), B(6, 6) and C(6, 3).
Triangle Q has vertices A'(2, −4), B'(0, −6) and C'(0, 0).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 1, x-step from A' to B' = −2
Scale factor = $-2 \div 1 = -2$
The centre C satisfies: $C + -2(A - C) = A'$, so $(1 - -2)C = A' - -2A$
x: $3 \times x = 2 - (-2) \times 5 = 12$, so $x = 4$
y: $3 \times y = (-4) - (-2) \times 5 = 6$, so $y = 2$
Answer: Enlargement, scale factor −2, centre (4, 2)

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6 Reverse percentages

The number of members of a club is decreased by 5% in one year.
The next year the number of members is increased by 40%.
There are now 532 members.
How many members were there to begin with?

Multipliers: 0.95, 1.4
Overall multiplier = 0.95 × 1.4 = 1.33
Original = final ÷ multiplier = 532 ÷ 1.33
= 400
Answer: 400

7 Rearranging when the subject appears twice

Make r the subject of $P(r + 4) = r - P$

Expand the bracket: $Pr + 4P = r - P$
Collect the r terms: $Pr - r = -4P - P$
Factorise: $r(P - 1) = -5P$
$r = \frac{5P}{1 - P}$
Answer: $\mathbf{r = \frac{5P}{1 - P}}$

8 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
20 of them had a mass between 0 and 10 kg.
(a) How many had a mass between 20 and 40 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
0 to 10: bar is 2 squares high and 10 wide, area 20, which represents 20
So 1 unit of area represents 20 ÷ 20 = 1
(a) 20 to 40: 1 × 20 = 20 units of area, so 20 × 1 = 20
(b) Total area = 20 + 20 + 20 + 30 = 90, so total = 90 × 1 = 90
Answer: (a) 20 (b) 90

9 Surds with unknowns

Given that $(a + \sqrt{2})(b + \sqrt{2}) = 14 + 7\sqrt{2}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{2})(b + \sqrt{2}) = ab + (a + b)\sqrt{2} + 2$
Compare the $\sqrt{2}$ terms: $a + b = 7$
Compare the integer terms: $ab + 2 = 14$, so $ab = 12$
Two numbers with sum 7 and product 12 are 3 and 4
Answer: a = 3, b = 4

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10 Perpendicular bisector

A is the point $(-7, 3)$ and B is the point $(3, -1)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Midpoint of AB = $\left(\frac{-7+3}{2}, \frac{3+(-1)}{2}\right) = (-2, 1)$

Gradient of AB = $\frac{-1-3}{3-(-7)} = -\frac{2}{5}$

The perpendicular gradient is $-1 \div \left(-\frac{2}{5}\right) = \frac{5}{2}$

Using the midpoint: $y - 1 = \frac{5}{2}(x + 2)$

$y = \frac{5}{2}x + 6$

Answer: $y = \frac{5}{2}x + 6$