Day 137 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 21:35 (London time).
It takes 13 hours 30 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 21:35 + 13 h 30 min = 11:05 the next day
Sydney is 9 hours ahead of London, so add 9 hours
20:05 the next day
Answer: 20:05 the next day

2 Mean of a changing set

The mean of 12 numbers is 136.
One number is removed and the mean of the remaining 11 numbers is 148.
Work out the number that was removed.

Total of the 12 numbers = 12 × 136 = 1632
Total of the 11 remaining = 11 × 148 = 1628
Number removed = 1632 − 1628 = 4
Answer: 4

3 Radius from chord and height of arc

A circular arc has a chord AB of length 40 cm.
The greatest distance between the chord and the arc is 4 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 4
AM = 40 ÷ 2 = 20. Pythagoras in triangle OMA: $r^2 = (r - 4)^2 + 20^2$
$r^2 = r^2 - 8r + 16 + 400$
8r = 416, so r = 52 cm
Answer: 52 cm

4 Rationalising the denominator

Write $\frac{14}{2 - \sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are integers.

Multiply top and bottom by $(2 + \sqrt{2})$:
$$\frac{14}{2 - \sqrt{2}} = \frac{14(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})}$$
Denominator: 4 − 2 = 2
$= 14 + 7\sqrt{2}$
Answer: $14 + 7\sqrt{2}$

5 Winning a match

Tariq and Rhys play a match. The first to win 2 chess games wins the match.

The probability that Tariq wins any one of the chess games is $\frac{1}{3}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Tariq wins then loses) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

P(Tariq loses then wins) = $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Total = $2 \times \frac{2}{9} = \frac{4}{9}$

Answer: $\frac{4}{9}$

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6 LCM problems

A red light flashes every 8 minutes.
A blue light flashes every 9 minutes.
A green light flashes every 24 minutes.
They all flash together at 10:00 am.
(a) At what time do they next flash together?

Find the LCM of 8, 9 and 24
$8 = 2^3$, $9 = 3^2$, $24 = 2^3 \times 3$
LCM = $2^3 \times 3^2 = 72$
72 minutes after 10:00 am is 11:12 am
Answer: 11:12 am

7 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 7 : 3.
Then 12 cats are removed and 18 more dogs are added.
The ratio is now 4 : 3.
How many dogs were there at the start?

Let the numbers at the start be 7k and 3k
(7k − 12) : (3k + 18) = 4 : 3, so 3(7k − 12) = 4(3k + 18)
21k − 36 = 12k + 72
9k = 108, so k = 12
Number of dogs at the start: 3 × 12 = 36
Answer: 36

8 Equations with algebraic fractions

Solve $\frac{4}{x-5} - \frac{3}{x-3} = \frac{1}{2}$

Multiply every term by (x − 5)(x − 3): $4(x-3) - 3(x-5) = \frac{1}{2} \times (x-5)(x-3)$
Multiply by 2 to clear the fraction: $8(x-3) - 6(x-5) = (x-5)(x-3)$
Expand and rearrange so that one side is 0: $x^2 - 10x + 9 = 0$
Factorise: $(x-9)(x-1) = 0$
x = 1 or x = 9
Answer: x = 1 or x = 9

9 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 5**a** + 4**b**, OQ = −**a** + 7**b** and OR = −5**a** + 9**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = −6**a** + 3**b**
QR = OR − OQ = −4**a** + 2**b**
QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

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10 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.

Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -5$ and $x = 2$, so $y = a(x + 5)(x - 2)$
When $x = 0$, $y = -20$: $a \times 5 \times (-2) = -20$, so $a = 2$
$y = 2(x + 5)(x - 2)$
$y = 2x^2 + 6x - 20$

Answer: $y = 2x^2 + 6x - 20$