Day 136 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 150°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 150° = 30°
Number of sides = 360° ÷ 30° = 12
Answer: 12

2 Powers of standard form

Work out the square root of 8.41×10^{12}.
Give your answer in standard form.

$8.41 = 2.9 \times 2.9$
$\sqrt{(10^{12})} = 10^{6}$
$\sqrt{(8.41 \times 10^{12})} = 2.9 \times 10^{6}$
Answer: 2.9×10^{6}

3 Exchange rates

Dina changes £400 into euros at £1 = €1.15.
Dina spends €180 and then changes the rest back into pounds at £1 = €1.4.
How many pounds does Dina get back?

£400 × 1.15 = €460
460 − 180 = €280
280 ÷ 1.4 = £200
Answer: £200

4 Area of a segment

The circle has centre O and radius 4 cm. A and B are points on the circle and angle AOB = 120°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{120}{360} \times \pi \times 4^2 = 16.755$ cm^2
Area of triangle OAB = $\frac{1}{2} \times 4 \times 4 \times \sin 120° = 6.9282$ cm^2
Segment = sector − triangle = 16.755 − 6.9282 = 9.8270
= 9.83 cm^2 (3 s.f.)
Answer: 9.83 cm^2

5 Inverse proportion in context

10 painters can paint a hall in 8 days.
After 5 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 10 × 8 = 80 painter-days
Work done = 10 × 5 = 50, so work left = 30
Workers left = 10 − 5 = 5
Time = 30 ÷ 5 = 6 days
Answer: 6 days

6 Transformations of graphs

The point P(12, −4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(−x) (b) y = f(x) − 3

(a) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (−12, −4)
(b) y = f(x) − 3 is a translation of 3 down: (12, −7)
Answer: (a) (−12, −4) (b) (12, −7)

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7 Proof with indices

Simplify fully $\frac{7^{n+2} - 7^n}{48}$

$7^{n+2} - 7^n = 7^n(7^2 - 1) = 7^n \times 48$

Divide by 48: $\frac{48 \times 7^n}{48} = 7^n$

Answer: 7^n

8 Laws of indices

Write $\frac{16^{n+1} \times 8^{n-2}}{16^n}$ in the form 2^{an+c}

where a and c are integers.

$16 = 2^4$, $8 = 2^3$, $16 = 2^4$

$16^{n+1} = 2^{4(n+1)}$, $8^{n-2} = 2^{3(n-2)}$, $16^n = 2^{4n}$

Exponent: $4(n + 1) + 3(n - 2) - 4n = 3n - 2$

Answer: a = 3, c = −2

9 Histograms

The histogram shows the lengths of leaves collected from a tree.

(a) Work out the total number of leaves.

(b) Estimate the number of leaves with a length between 40 cm and 55 cm.

Frequency = frequency density × class width

20 to 30: 1.4 × 10 = 14

30 to 50: 1.8 × 20 = 36

50 to 60: 1.6 × 10 = 16

60 to 80: 1.4 × 20 = 28

80 to 100: 1 × 20 = 20

(a) Total = 114

(b) 40 to 50: 1.8 × 10 = 18

50 to 55: 1.6 × 5 = 8, so estimate = 26

Answer: (a) 114 (b) 26

10 Inverse functions

$f(x) = \frac{7}{x - 4} + 3$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(-4)$.

(a) Let $y = \frac{7}{x - 4} + 3$, so $y - 3 = \frac{7}{x - 4}$

Make x the subject: $x - 4 = \frac{7}{y - 3}$

$x = \frac{7}{y - 3} + 4$

$f^{-1}(x) = \frac{7}{x - 3} + 4$

(b) $f^{-1}(-4) = \frac{7}{-4 - 3} + 4 = 3$

Answer: (a) $f^{-1}(x) = \frac{7}{x - 3} + 4$ (b) 3