Day 135 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 168°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 168° = 12°
Number of sides = 360° ÷ 12° = 30
Answer: 30

2 HCF and LCM

$A = 2^2 \times 7^2 \times 11$
$B = 7^2 \times 11$
Write down, as a product of prime factors,
(a) the HCF of A and B
(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers
$= 7^2 \times 11$
LCM: take the highest power of every prime that appears
$= 2^2 \times 7^2 \times 11$
Answer: (a) $7^2 \times 11$ (b) $2^2 \times 7^2 \times 11$

3 Mean of a changing set

The mean of 9 numbers is 105.
A new number is added and the mean of all 10 numbers is 96.8.
Work out the new number.

Total of the 9 numbers = 9 × 105 = 945
New total = 10 × 96.8 = 968
New number = 968 − 945 = 23
Answer: 23

4 Growth and decay

A culture has 1000 bacteria.
The number of bacteria increases by 8% each hour.
After how many whole hours will there first be more than 1350 bacteria? (Calculator)

Multiplier each hour = 1.08
After 3 hours: $1000 \times 1.08^3 = 1260$
After 4 hours: $1000 \times 1.08^4 = 1360$
So the first time it is more than 1350 is after 4 hours
Answer: 4 hours

5 Enlargement

Triangle T has vertices A(2, −6), B(5, −4), C(1, −1).
T is enlarged with scale factor 3 and centre (4, −3).
(a) Write down the coordinates of the image of A.
(b) The area of T is 8 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (2 − 4, −6 − (−3)) = (−2, −3)
Multiply by 3: (−6, −9)
Image of A = centre + this vector = (−2, −12)
(b) Area scale factor = $(3)^2 = 9$
Area = 8 × 9 = 72 square units
Answer: (a) (−2, −12) (b) 72

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6 Simple and compound interest

Evan invests £1,000 for 4 years at 5% per year simple interest.
Omar invests £1,000 for 4 years at 5% per year compound interest.
Work out how much more interest Omar earns than Evan.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 5% × 4 = £200
Compound: £1,000 × 1.05^4 = £1,215.51
Compound interest = £1,215.51 − £1,000 = £215.51
Difference = £215.51 − £200 = £15.51
Answer: £15.51

7 Completing the square

(a) Write $x^2 - 4x + 12$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 4x + 12$

(a) Half of −4 is −2, so $x^2 - 4x = (x - 2)^2 - 4$
$x^2 - 4x + 12 = (x - 2)^2 - 4 + 12$
$= (x - 2)^2 + 8$
(b) The minimum is when $x - 2 = 0$, so $x = 2$ and $y = 8$
Answer: (a) $(x - 2)^2 + 8$ (b) (2, 8)

8 Rearranging when the subject appears twice

Make x the subject of $y(x + 4) = x - y$

Expand the bracket: $yx + 4y = x - y$
Collect the x terms: $yx - x = -4y - y$
Factorise: $x(y - 1) = -5y$
$x = \frac{5y}{1 - y}$
Answer: $x = \frac{5y}{1 - y}$

9 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 30 km on a bearing of 240° to Q. (Calculator)
It then travels 99 km on a bearing of 165° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 240° − 180° = 060°
Angle PQR = 105° (the difference between the bearings 060° and 165°)
$PR^2 = 30^2 + 99^2 - 2 \times 30 \times 99 \times \cos 105° = 12238$, so PR = 110.6 km
(b) $\cos(QPR) = \frac{30^2 + PR^2 - 99^2}{2 \times 30 \times PR}$, so angle QPR = 59.8°
R is anticlockwise of the direction PQ, so the bearing is 240° − 60° = 180°
Answer: (a) 110.6 km (b) 180°

10 Density of an alloy

An alloy contains 355 g of zinc and 89 g of nickel.
The density of zinc is 7.1 g/cm^3. The density of nickel is 8.9 g/cm^3.
Work out the density of the alloy.

Volume of zinc = 355 ÷ 7.1 = 50 cm^3
Volume of nickel = 89 ÷ 8.9 = 10 cm^3
Total mass = 444 g, total volume = 60 cm^3
Density = 444 ÷ 60 = 7.4 g/cm^3
Answer: 7.4 g/cm^3