Day 134 **S Tier 10-a-day: Model Solutions**

1 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 60°
tan 60° = $\sqrt{3}$
x = 6 × tan 60° = 6 × $\sqrt{3}$
x = $6\sqrt{3}$
Answer: x = $6\sqrt{3}$

2 Factorising quadratics

Factorise $3x^2 + 2x - 8$

3 × (−8) = −24. Two numbers that multiply to −24 and add to 2 are 6 and −4
Split the middle term: $3x^2 + 6x - 4x - 8$
Factorise in pairs: $3x(x + 2) - 4(x + 2)$
$= (3x - 4)(x + 2)$
Answer: $(3x - 4)(x + 2)$

3 Pressure

A wooden block in the shape of a cuboid weighs 990 N.
Its smallest face has area 100 cm^2. Its largest face has area 1,200 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 1,200 cm^2 = 0.12 m^2
(a) Greatest pressure is on the smallest face: 990 ÷ 0.01 = 99,000 N/m^2
(b) Least pressure is on the largest face: 990 ÷ 0.12 = 8,250 N/m^2
Answer: (a) 99,000 N/m^2 (b) 8,250 N/m^2

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 78°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 78 = 102°
(b) The angle at the circumference is half the angle at the centre: 102 ÷ 2 = 51°
Answer: (a) 102° (b) 51°

5 LCM problems

A red light flashes every 6 minutes.
A blue light flashes every 12 minutes.
A green light flashes every 18 minutes.
They all flash together at 10:00 am.
(a) At what time do they next flash together?

Find the LCM of 6, 12 and 18
6 = 2 × 3, 12 = 2^2 × 3, 18 = 2 × 3^2
LCM = $2^2 \times 3^2$ = 36
36 minutes after 10:00 am is 10:36 am
Answer: 10:36 am

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6 Recasting solids

A solid metal sphere of radius 12 cm is melted down and recast into a solid cylinder of radius 8 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 12^3 = 2304\pi$

Volume of cylinder = $\pi \times 8^2 \times h = 64\pi h$
$64h = 2304$
$h = 36$ cm
Answer: 36 cm

7 Quadratic inequalities

Solve $3x^2 + 15x \leq 0$

Divide by 3: $x^2 + 5x \leq 0$
Solve $x^2 + 5x = 0$: $(x + 5)(x) = 0$, so $x = -5$ or $x = 0$
The graph is a U shape crossing the x-axis at -5 and 0; it is below the axis between the roots
$-5 \leq x \leq 0$
Answer: $-5 \leq x \leq 0$

8 Functions that are their own inverse

$f(x) = \frac{2x + 3}{x - 2}$ where $x \neq 2$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 2)$
$ff(x) = \frac{2(2x + 3) + 3(x - 2)}{(2x + 3) - 2(x - 2)}$
Top = $4x + 6 + 3x - 6 = 7x$
Bottom = $2x + 3 - 2x + 4 = 7$
$ff(x) = \frac{7x}{7} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{2x + 3}{x - 2}$

Answer: (b) $f^{-1}(x) = \frac{2x + 3}{x - 2}$

9 Exact lengths and areas

A rectangle has area $(6 + 5\sqrt{3})$ cm^2 and length $(3 + 2\sqrt{3})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length = $\frac{6 + 5\sqrt{3}}{3 + 2\sqrt{3}}$

Multiply top and bottom by $(3 - 2\sqrt{3})$:
Numerator: $(6 + 5\sqrt{3})(3 - 2\sqrt{3}) = -12 + 3\sqrt{3}$
Denominator: $9 - 12 = -3$
Width = $4 - \sqrt{3}$
Answer: $4 - \sqrt{3}$ cm

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10 Combining means

The mean of the marks in a test for the men is 67.
The mean of the marks in a test for the women is 60.
The mean for all of the men and women together is 62.8.
Work out the ratio of the number of men to the number of women.

Let there be p men and q women
Total = 67p + 60q, and the overall mean gives 67p + 60q = 62.8(p + q)
4.2p = 2.8q
p : q = 2.8 : 4.2 = 2 : 3
Answer: 2 : 3