Day 133 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 171°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 171° = 9°
Number of sides = 360° ÷ 9° = 40
Answer: 40

2 Factorising quadratics

Factorise $12x^2 - 17x - 5$

12 × (−5) = −60. Two numbers that multiply to −60 and add to −17 are −20 and 3
Split the middle term: $12x^2 - 20x + 3x - 5$
Factorise in pairs: $4x(3x - 5) + (3x - 5)$
$= (4x + 1)(3x - 5)$
Answer: (4x + 1)(3x − 5)

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 20°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 20°
Angle COB is an exterior angle of triangle OAC: 20 + 20 = 40°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 40°

4 Speed and a moving train

A train travels at a constant speed of 54 km/h.
It takes 60 seconds to cross completely over a bridge which is 720 m long.
Work out the length of the train, in metres.

Speed = 54 km/h = 54,000 ÷ 3600 = 15 m/s
Distance travelled in 60 s = 15 × 60 = 900 m
This is the length of the bridge plus the length of the train
Train = 900 − 720 = 180 m
Answer: 180 m

5 Percentages of percentages

In a group of employees, 70% are men and the rest are women.
30% of the men and 15% of the women work from home.
What percentage of all the employees work from home?

Take 100 employees: 70 are men and 30 are women
30% of 70 = 21 and 15% of 30 = 4.5
21 + 4.5 = 25.5 out of 100
Answer: 25.5%

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6 Direct proportion formulae

E is directly proportional to the square root of v.
When v = 1, E = 5.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 30.

(a) $E = k\sqrt{v}$
$5 = k \times 1$, so $k = 5 \div 1 = 5$
$E = 5\sqrt{v}$
(b) $30 = 5\sqrt{v}$, so $\sqrt{v} = 30 \div 5 = 6$, giving $v = 36$
Answer: (a) $E = 5\sqrt{v}$ (b) $v = 36$

7 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 40 cm. (Calculator)
The outer radius is 6 cm and the inner radius is 4 cm.
The density of the metal is 8.5 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal $= \pi \times 6^2 \times 40 - \pi \times 4^2 \times 40 = \pi(36 - 16) \times 40$
$= 2513.3$ cm^3
Mass = density × volume $= 8.5 \times 2513.3 = 21363$ g
= 21.4 kg (3 s.f.)
Answer: 21.4 kg

8 Proof with indices

Show that $2^{t+2} - 2^t$ is a multiple of 3 for all integer values of t.

$2^{t+2} = 2^t \times 2^2$
$2^{t+2} - 2^t = 2^t(2^2 - 1)$
$2^2 - 1 = 3$
$= 3 \times 2^t$, which is a multiple of 3
Answer: 3×2^t

9 Functions that are their own inverse

$f(x) = \frac{-2x + 4}{3x + 2}$ where $x \neq -\frac{2}{3}$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (3x + 2)
$ff(x) = \frac{(-2)(-2x + 4) + 4(3x + 2)}{3(-2x + 4) - (-2)(3x + 2)}$
Top $= 4x - 8 + 12x + 8 = 16x$
Bottom $= -6x + 12 + 6x + 4 = 16$
$ff(x) = \frac{16x}{16} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-2x + 4}{3x + 2}$

Answer: (b) $f^{-1}(x) = \frac{-2x + 4}{3x + 2}$

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10 Fraction problems

A bag contains counters.

$\frac{1}{8}$ of the counters are red and $\frac{3}{8}$ of the counters are blue.

The rest of the counters, 32 of them, are green.

How many blue counters are there?

Fraction that are green = $1 - \frac{1}{8} - \frac{3}{8} = \frac{1}{2}$

$\frac{1}{2}$ of the total is 32

Total = $32 \div \frac{1}{2} = 64$

Blue counters = $\frac{3}{8}$ of $64 = 24$

Answer: 24