Day 132 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 25 is taken. Work out how many come from Year 9.
(b) A different stratified sample has 8 from Year 10. Work out the size of this sample.

Total = 110 + 30 + 70 + 40 = 250

(a) Year 9: $\frac{70}{250} \times 25 = 7$

(b) Fraction sampled = $8 \div 40 = \frac{1}{5}$, so sample size = $\frac{1}{5} \times 250 = 50$

Answer: (a) 7 (b) 50

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (2x + 5)°
Angle PQR = (x + 25)°
Angle PRS = (x + 58)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
x + 58 = 3x + 30
2x = 28
x = 14
(b) Angle PRS = 72° so angle PRQ = 180° − 72° = 108°
Answer: (a) x = 14 (b) 108°

3 Inverse proportion in context

9 machines can make a batch of bolts in 30 hours.
After 6 hours, 3 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 9 × 30 = 270 machine-hours
Work done = 9 × 6 = 54, so work left = 216
Workers left = 9 − 3 = 6
Time = 216 ÷ 6 = 36 hours
Answer: 36 hours

4 Inequalities

(a) Solve $8 - 2x \geq 2$
(b) n is an integer. Write down all the values of n that satisfy
$5 \leq 2n + 3 \leq 11$

(a) Subtract 8: $-2x \geq -6$
Divide by −2 and reverse the inequality sign: $x \leq 3$
(b) Subtract 3: $2 \leq 2n \leq 8$
Divide by 2: $1 \leq n \leq 4$
Integers: 1, 2, 3, 4
Answer: (a) $x \leq 3$ (b) 1, 2, 3, 4

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5 Recurring decimals

Convert the recurring decimal $0.9\dot{2}\dot{0}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.9202...
10x = 9.2020...
1000x = 920.2020...
Subtract: 990x = 911
$x = \frac{911}{990}$

Answer: $\mathbf{\frac{911}{990}}$

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 75° and angle CAQ = 59°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 59° (alternate segment theorem)
(b) Angle ACB = angle PAB = 75° (alternate segment theorem)
Answer: (a) 59° (b) 75°

7 Tree diagrams without replacement

A bag contains 5 red counters, 6 blue counters and 1 green counter.
Two counters are taken at random without replacement.
Work out the probability that (a) both are the same colour, (b) exactly one is green.

(a) Add the ways: $\frac{5}{12}\times\frac{4}{11} + \frac{6}{12}\times\frac{5}{11} = \frac{25}{66}$

(b) P(exactly one green) = $\frac{1}{12}\times\frac{11}{11} + \frac{11}{12}\times\frac{1}{11} = \frac{1}{6}$

Answer: (a) $\mathbf{\frac{25}{66}}$ **(b)** $\mathbf{\frac{1}{6}}$

8 Bounds of area and perimeter

A rectangle has length 25 cm and width 6 cm.
Both measurements are correct to the nearest centimetre.
Evan says that the area of the rectangle could be exactly 150 cm^2.
Is Evan correct? You must show your working.

$24.5 \leq$ length < 25.5 and $5.5 \leq$ width < 6.5
Lower bound of area = 24.5 × 5.5 = 134.75
Upper bound of area = 25.5 × 6.5 = 165.75
So $134.75 \leq$ area < 165.75
150 is in this interval, so Evan is correct
Answer: Yes, 150 is between 134.75 and 165.75

9 Algebraic proof (harder)

Prove algebraically that $m^3 + 11m$ is a multiple of 6 for all integer values of m.

$m^3 + 11m = (m^3 - m) + 12m = (m - 1)m(m + 1) + 12m$
$(m - 1)m(m + 1)$ is the product of three consecutive integers, so it contains a multiple of 2 and a multiple of 3, so it is a multiple of 6
12m = 6 × 2m is also a multiple of 6
The sum of two multiples of 6 is a multiple of 6
Answer: Proved

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10 Pythagoras with algebra

ABC is a right-angled triangle with sides $(2x - 1)$ cm, $(x + 2)$ cm and $(2x + 1)$ cm. The longest side is $(2x + 1)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(2x - 1)^2 + (x + 2)^2 = (2x + 1)^2$
Expand and collect terms: $x^2 - 4x + 4 = 0$
Factorise: $(x - 2)^2 = 0$
$x = 2$
(b) The two shorter sides are 3 cm and 4 cm
Area $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (a) x = 2 (b) 6 cm^2