Day 131 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The sum of the interior angles of a polygon is 2880°.
Work out the number of sides of the polygon.

Sum of interior angles = $(n - 2) \times 180°$
$(n - 2) \times 180 = 2880$
$n - 2 = 2880 \div 180 = 16$
$n = 18$
Answer: 18

2 Inverse proportion in context

18 machines can make a batch of bolts in 30 hours.
After 4 hours, 6 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = $18 \times 30 = 540$ machine-hours
Work done = $18 \times 4 = 72$, so work left = 468
Workers left = $18 - 6 = 12$
Time = $468 \div 12 = 39$ hours
Answer: 39 hours

3 Composite functions

$f(x) = 3x - 1$ and $g(x) = x^2 - 1$
(a) Find $gf(x)$. Give your answer in the form $ax^2 + bx + c$.
(b) Solve $gf(x) = 195$

(a) $gf(x) = g(3x - 1) = (3x - 1)^2 - 1$
$= 9x^2 - 6x$
(b) $(3x - 1)^2 = 196$, so $3x - 1 = \pm 14$
$3x - 1 = 14$ or $3x - 1 = -14$
$x = -\frac{13}{3}$ or $x = 5$

Answer: (a) $9x^2 - 6x$ (b) $x = -\frac{13}{3}$ or $x = 5$

4 Average speed

A cyclist travels the first half of a journey at 20 km/h.
It travels the second half of the same journey at 80 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 80 km (a common multiple of 20 and 80)
Time for first half = $80 \div 20 = 4$ hours
Time for second half = $80 \div 80 = 1$ hours
Average speed = total distance ÷ total time = $160 \div 5 = 32$ km/h
Answer: 32 km/h

5 Perpendicular bisector

A is the point $(-5, -4)$ and B is the point $(3, -2)$.
Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(-1, -3)$
Gradient of AB = $\frac{1}{4}$
Perpendicular gradient = -4
$y - (-3) = (-4)(x - (-1))$
Rearrange to integer coefficients: $4x + y = -7$
Answer: $4x + y = -7$

Day 131 **S Tier 10-a-day: Model Solutions**

6 Compound interest

Uma invests £10,000.
The account pays compound interest of 4% a year for the first 2 years,
then 1% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 2 years, then 1.01 for 2 years
£10,000 × 1.04^2 × 1.01^2
= £11,033.40
Answer: £11,033.40

7 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 2) cm, (x + 9) cm and (2x + 5) cm. The longest side is (2x + 5) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 2)^2 + (x + 9)^2 = (2x + 5)^2$
Expand and collect terms: $2x^2 - 2x - 60 = 0$
Factorise: $2(x - 6)(x + 5) = 0$
x = 6 or x = −5, and x must be positive, so x = 6
(b) The two shorter sides are 8 cm and 15 cm
Area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2
Answer: (a) x = 6 (b) 60 cm^2

8 Equal roots and real roots

The equation $kx^2 - 7x + 1 = 0$ has equal roots.
Find the value of k.

Equal roots means $b^2 - 4ac = 0$
$(-7)^2 - 4 \times k \times 1 = 0$, so $49 = 4k$
$k = \frac{49}{4}$
Answer: k = $\frac{49}{4}$

9 Surds and fractions

Simplify $\frac{1}{3 + \sqrt{3}} - \frac{1}{3 - \sqrt{3}}$
Give your answer with a rational denominator.

Common denominator: $(3 + \sqrt{3})(3 - \sqrt{3}) = 9 - 3 = 6$
Numerator: $(3 - \sqrt{3}) - (3 + \sqrt{3})$
$= -2\sqrt{3}$
$\frac{-2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}$
Answer: $-\frac{\sqrt{3}}{3}$

Day 131 **S Tier 10-a-day: Model Solutions**

10 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 100 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a time greater than 55.

(a) Median at cumulative frequency 50, so median = 40
Lower quartile at 25, so LQ = 30. Upper quartile at 75, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 55, cumulative frequency = 81
Number greater = 100 − 81 = 19
Answer: (a) median 40, IQR 20 (b) 19