Day 130 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 26 is taken. Work out how many come from Group D.
(b) A different stratified sample has 9 from Group A. Work out the size of this sample.

Total = 75 + 175 + 250 + 150 = 650

(a) Group D: $\frac{150}{650} \times 26 = 6$

(b) Fraction sampled = $9 \div 75 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 650 = 78$

Answer: (a) 6 (b) 78

2 Equations with fractions

Solve $\frac{x-3}{4} + \frac{x-3}{2} = 3$

Multiply every term by 4: $(x - 3) + 2(x - 3) = 12$
Expand: $3x - 9 = 12$
$3x = 21$
$x = 7$
Answer: x = 7

3 Prime factors and indices

Write $18^3 \times 15^3$ as a product of powers of prime numbers.

$18 = 2 \times 3^2$, $15 = 3 \times 5$
$18^3 = 2^3 \times 3^6$
$15^3 = 3^3 \times 5^3$
Add the powers of each prime: $2^3 \times 3^9 \times 5^3$
Answer: $2^3 \times 3^9 \times 5^3$

4 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 16^2 + 8^2 = 320$
$AG^2 = AC^2 + CG^2 = 320 + 2^2 = 324$
AG = 18 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{2}{\sqrt{320}}$

angle GAC = 6.4° (1 d.p.)
Answer: (a) 18 cm (b) 6.4°

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5 Fractions of fractions

Priya read $\frac{3}{10}$ of a book on Saturday.

On Sunday she read $\frac{6}{7}$ of the pages that were left.

She still had 35 pages to read.
How many pages are in the book?

After the first part, $\frac{7}{10}$ remains

After the second, $\frac{1}{7}$ of $\frac{7}{10}$ remains = $\frac{1}{10}$ of the total

$\frac{1}{10}$ of the total is 35

Total = $35 \div \frac{1}{10} = 350$

Answer: 350 pages

6 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 8 : 5.

Then $\frac{3}{4}$ of the blue and $\frac{1}{4}$ of the green are taken out.

Work out the ratio of blue to green now.

Left: $\frac{1}{4} \times 8 = 2$ of the blue part, $\frac{3}{4} \times 5 = \frac{15}{4}$ of the green part

$2 : \frac{15}{4}$

Multiply by 4: 8 : 15

Answer: 8 : 15

7 Percentages with an unknown rate

The price of a car is decreased by x%.
The new price is then increased by x%.
The final price is 4% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 4% less, so the multiplier is 0.96: $1 - \frac{x^2}{10000} = 0.96$

$\frac{x^2}{10000} = 0.04$, so $x^2 = 400$ and $x = 20$

Answer: x = 20

8 Terms of a quadratic sequence

The nth term of a sequence is $n^2 - 3n + 3$.
(a) Work out the 7th term.
(b) Is 13 a term of the sequence? You must show your working.

(a) $7^2 - 3 \times 7 + 3 = 31$

(b) Solve $n^2 - 3n + 3 = 13$, so $n^2 - 3n - 10 = 0$
Factorise or use the formula: $n = 5$ or $n = -2$
n = 5 is a positive whole number, so 13 is term number 5

Answer: (a) 31 (b) Yes, it is the 5th term

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9 Joint and inverse variation

F is directly proportional to m and inversely proportional to the square of d.
When m = 6 and d = 3, F = 8.
Work out the value of F when m = 2 and d = 2.

$F = \frac{km}{d^2}$

$8 = \frac{k \times 6}{9}$, so $k = 8 \times 9 \div 6 = 12$

$F = \frac{12 \times 2}{4} = 6$

Answer: F = 6

10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 1 cm, top radius 6 cm and base radius 9 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+1} = \frac{6}{9}$

$9h = 6(h + 1)$, so $3h = 6$ and $h = 2$

Large cone: $\frac{1}{3} \times \pi \times 9^2 \times 3 = 81\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 2 = 24\pi$

Frustum = $81\pi - 24\pi = 57\pi$ cm^3

Answer: 57π cm^3