Day 13 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

FitLife has a joining fee of £7.50 plus £1 per month.
GymNow has a joining fee of £5 plus £1.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 10 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when 7.50 + n = 5 + 1.50n
0.50n = 2.50, so n = 5
(b) FitLife: £7.50 + £1 × 10 = £17.50
GymNow: £5 + £1.50 × 10 = £20
FitLife is cheaper by £2.50
Answer: (a) 5 (b) FitLife, by £2.50

2 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 45°

$\cos 45° = \frac{\sqrt{2}}{2}$

$x = 7 \times \cos 45° = 7 \times \frac{\sqrt{2}}{2}$

$x = \frac{7\sqrt{2}}{2}$

Answer: $\mathbf{x = \frac{7\sqrt{2}}{2}}$

3 Quadratic formula and surds

Solve $4x^2 + 7x - 4 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 4, b = 7, c = −4

$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 4 \times (-4)}}{2 \times 4}$

$x = \frac{-7 \pm \sqrt{113}}{8}$

Answer: $\mathbf{x = \frac{-7 \pm \sqrt{113}}{8}}$

4 Column vectors

$\mathbf{a} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.
Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 6 + 3k \\ 0 + 1k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{6 + 3k}{3} = \frac{0 + 1k}{3}$

3(6 + 3k) = 3(0 + 1k)
k × (6) = −18
k = −3
Answer: k = −3

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5 Gradient of a chord

A and B are points on the curve $y = x^3 - 5x$.
The x coordinate of A is -2 and the x coordinate of B is 1.
Work out the gradient of the line AB.

At A: y = 2. At B: y = -4
A is $(-2, 2)$ and B is $(1, -4)$
Gradient = $\frac{-4 - 2}{1 - (-2)} = \frac{-6}{3} = -2$

Answer: -2

6 Average speed

A cyclist travels the first half of a journey at 30 km/h.
It travels the second half of the same journey at 45 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 90 km (a common multiple of 30 and 45)
Time for first half = 90 ÷ 30 = 3 hours
Time for second half = 90 ÷ 45 = 2 hours
Average speed = total distance ÷ total time = 180 ÷ 5 = 36 km/h

Answer: 36 km/h

7 Profit and loss

A retailer buys a guitar for £160.
It adds an 80% mark-up to get the selling price.
In a sale the selling price is reduced by 10%.
Work out the percentage profit or loss on the original cost.

Marked price = £160 × 1.8 = £288
Sale price = £288 × 0.9 = £259.20
Profit = £99.20

Percentage = $\frac{99.2}{160} \times 100 = 62\%$

Answer: 62% profit

8 Laws of indices

Write $\frac{4^{n+3} \times 16^{n+2}}{4^n}$ in the form 2^{an+c}

where a and c are integers.

$4 = 2^2$, $16 = 2^4$, $4 = 2^2$
$4^{n+3} = 2^{2(n+3)}$, $16^{n+2} = 2^{4(n+2)}$, $4^n = 2^{2n}$
Exponent: $2(n + 3) + 4(n + 2) - 2n = 4n + 14$

Answer: a = 4, c = 14

9 Mixtures and percentage profit

A shop mixes two types of mixed nuts.
Type A costs £13.00 per kg and type B costs £7.00 per kg.
They are mixed in the ratio A : B = 1 : 3.
The mixture is sold for £9.35 per kg.
Work out the percentage profit.

Cost of 4 kg of mixture = 1 × £13.00 + 3 × £7.00 = £34.00
Cost per kg = £34.00 ÷ 4 = £8.50
Profit per kg = £9.35 − £8.50 = £0.85
Percentage profit = 0.85 ÷ 8.50 × 100 = 10%

Answer: 10%

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10 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 32°.
B is a point on the same line, 20 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 17°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 32° = \frac{h}{x}$, so $x = h \div \tan 32°$

From B: $\tan 17° = \frac{h}{x + 20}$, so $x + 20 = h \div \tan 17°$

Subtract: $20 = h \div \tan 17° - h \div \tan 32° = h(3.2709 - 1.6003)$
$h = 20 \div 1.6705 = 12.0$ m (1 d.p.)
Answer: 12.0 m