Day 129 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $12x^2 + 37x + 28$

$12 \times 28 = 336$. Two numbers that multiply to 336 and add to 37 are 16 and 21

Split the middle term: $12x^2 + 16x + 21x + 28$

Factorise in pairs: $4x(3x + 4) + 7(3x + 4)$

$= (4x + 7)(3x + 4)$

Answer: $(4x + 7)(3x + 4)$

2 Probability from a frequency tree

A factory makes 5,000 parts. 4% of them are faulty.
A scanner flags 95% of faulty parts and wrongly flags 2% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

4% of 5,000 = 200 are faulty and 4,800 are not

Flagged and genuine: $200 \times 95\% = 190$

Flagged but not: $4{,}800 \times 2\% = 96$

Total flagged or positive = 190 + 96 = 286

Probability = $190 \div 286 = \frac{95}{143}$

Answer: $\frac{95}{143}$

3 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 2) cm, DE = 3 cm and BC = 9 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 2} = \frac{3}{9}$

$9x = 3(2x + 2)$

$9x = 6x + 6$

$3x = 6$

$x = 2$

(b) $AB = 2x + 2 = 6$ cm

Answer: (a) $x = 2$ (b) 6 cm

4 Quadratic inequalities

Solve $x^2 \leq -10x - 21$

Rearrange so that one side is 0: $x^2 + 10x + 21 \leq 0$

Solve $x^2 + 10x + 21 = 0$: $(x + 7)(x + 3) = 0$, so $x = -7$ or $x = -3$

The graph is a U shape crossing the x-axis at -7 and -3; it is below the axis between the roots

$-7 \leq x \leq -3$

Answer: $-7 \leq x \leq -3$

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5 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £5.00 per kg and type B costs £12.00 per kg.
They are mixed in the ratio A : B = 3 : 1.
The mixture is sold for £9.45 per kg.
Work out the percentage profit.

Cost of 4 kg of mixture = 3 × £5.00 + 1 × £12.00 = £27.00
Cost per kg = £27.00 ÷ 4 = £6.75
Profit per kg = £9.45 − £6.75 = £2.70
Percentage profit = 2.70 ÷ 6.75 × 100 = 40%
Answer: 40%

6 Standard form problems

A virus has a diameter of 2.5×10^{-7} metres.
Viruses are placed in a straight line, touching, to make a length of 9.5×10^{-1} metres.
How many viruses are in the line?
Give your answer in standard form.

Number = total length ÷ diameter
$(9.5 \times 10^{-1}) \div (2.5 \times 10^{-7})$
$9.5 \div 2.5 = 3.8$ and $10^{-1} \div 10^{-7} = 10^{6}$
$= 3.8 \times 10^{6}$
Answer: 3.8×10^{6}

7 Shaded areas: exact answers

ABCD is a square of side 8 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 8) = $\frac{1}{4} \times \pi \times 8^2 = 16\pi$
The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = $2 \times 16\pi - 64 = 32\pi - 64$
Answer: $(32\pi - 64)$ cm^2

8 Tangent to a circle

A circle has equation $(x + 2)^2 + (y + 1)^2 = 100$.
The point P(−10, 5) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (−2, −1). The gradient of the radius to P is $\frac{5 - (-1)}{-10 - (-2)} = -\frac{3}{4}$
The tangent is perpendicular to the radius, so its gradient is $\frac{4}{3}$. Using P: $y - 5 = \frac{4}{3}(x + 10)$
Multiply by 3: $3(y - 5) = 4(x + 10)$
Expand and rearrange: $-4x + 3y = 55$
Multiply by −1: $4x - 3y = -55$
Answer: $4x - 3y = -55$

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9 Loci and regions

ABCD is a rectangle with AB = 3 cm and AD = 7 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2

Answer: $\frac{1}{2}\pi$ cm^2

10 Nested surds

Show that $\sqrt{15 + 6\sqrt{6}} = 3 + \sqrt{6}$

Square the right-hand side: $(3 + \sqrt{6})^2 = 9 + 6\sqrt{6} + 6$
$= 15 + 6\sqrt{6}$
So $\sqrt{15 + 6\sqrt{6}} = 3 + \sqrt{6}$
Answer: $\sqrt{15 + 6\sqrt{6}} = 3 + \sqrt{6}$