Day 128 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 38°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 38°
Angle AOB = 180 − 2 × 38 = 104°
(b) The angle at the circumference is half the angle at the centre: 104 ÷ 2 = 52°
Answer: (a) 104° (b) 52°

2 Simplifying surds

Write $\sqrt{20} + \sqrt{180} - \sqrt{125}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{20} = 2\sqrt{5}$; $\sqrt{180} = 6\sqrt{5}$; $\sqrt{125} = 5\sqrt{5}$
$(2 + 6 - 5)\sqrt{5} = 3\sqrt{5}$
Answer: $3\sqrt{5}$

3 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 180}{x^2 - 14x + 48}$

Factorise the top: $5(x - 6)(x + 6)$
Factorise the bottom: $(x - 6)(x - 8)$
Cancel the common factor $(x - 6)$: $\frac{5(x + 6)}{x - 8}$
Answer: $\frac{5(x + 6)}{x - 8}$

4 Area of a triangle using sine

Triangle ABC has AB = 4 cm and BC = 12 cm.
The area of triangle ABC is $12\sqrt{2}$ cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$
$12\sqrt{2} = \frac{1}{2} \times 4 \times 12 \times \sin B$
$\sin B = 12\sqrt{2} \div 24 = \frac{\sqrt{2}}{2}$
$\sin 45° = \frac{\sqrt{2}}{2}$, and B is acute, so B = 45°
Answer: 45°

5 Average speed

A car travels the first half of a journey at 50 km/h.
It travels the second half of the same journey at 75 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 150 km (a common multiple of 50 and 75)
Time for first half = 150 ÷ 50 = 3 hours
Time for second half = 150 ÷ 75 = 2 hours
Average speed = total distance ÷ total time = 300 ÷ 5 = 60 km/h
Answer: 60 km/h

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6 Standard form problems

A factory makes 7.2×10^3 bottles each day.
It runs for 4×10^2 days.
How many bottles does it make?
Give your answer in standard form.

Total = $(7.2 \times 10^3) \times (4 \times 10^2)$
$7.2 \times 4 = 28.8$ and $10^3 \times 10^2 = 10^5$
$= 2.88 \times 10^6$
Answer: 2.88×10^6

7 Changing the subject

Make u the subject of $v = \frac{5}{u^2}$
where u is positive.

Multiply by u^2: $vu^2 = 5$
Divide by v: $u^2 = \frac{5}{v}$
Square root: $u = \sqrt{\frac{5}{v}}$
Answer: $u = \sqrt{\frac{5}{v}}$

8 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 67 km on a bearing of 180° to Q. (Calculator)
It then travels 85 km on a bearing of 090° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 180° − 180° = 000°
Angle PQR = 90° (the difference between the bearings 000° and 090°)
$PR^2 = 67^2 + 85^2 - 2 \times 67 \times 85 \times \cos 90° = 11714$, so PR = 108.2 km
(b) $\cos(QPR) = \frac{67^2 + PR^2 - 85^2}{2 \times 67 \times PR}$, so angle QPR = 51.8°
R is anticlockwise of the direction PQ, so the bearing is 180° − 52° = 128°
Answer: (a) 108.2 km (b) 128°

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to the square of t.
When v = 4 and t = 2, P = 48.
Work out the value of P when v = 5 and t = 2.

$P = \frac{kv^2}{t^2}$
$48 = \frac{k \times 16}{4}$, so $k = 48 \times 4 \div 16 = 12$
$P = \frac{12 \times 25}{4} = 75$
Answer: P = 75

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10 Tangency conditions

The line $y = 3x + k$ is a tangent to the curve $y = 3x^2 - 3x + 3$.
Find the value of k.

Equate: $3x^2 - 3x + 3 = 3x + k$

Rearrange: $3x^2 - 6x + 3 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $36 - 4 \times 3 \times (3 - k) = 0$

$12k = 0$

$k = 0$

Answer: k = 0