Day 127 **S Tier 10-a-day: Model Solutions**

1 Bounds of area and perimeter

A rectangle has length 9 cm and width 13 cm.
Both measurements are correct to the nearest centimetre.
Ivan says that the perimeter of the rectangle could be exactly 39 cm.
Is Ivan correct? You must show your working.

$8.5 \leq$ length < 9.5 and $12.5 \leq$ width < 13.5
Lower bound of perimeter $= 2 \times 8.5 + 2 \times 12.5 = 42$
Upper bound of perimeter $= 2 \times 9.5 + 2 \times 13.5 = 46$
So $42 \leq$ perimeter < 46
39 is outside this interval, so Ivan is not correct
Answer: No, 39 is outside the interval

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM $= 24 \div 2 = 12$ cm
$OA^2 = OM^2 + AM^2 = 5^2 + 12^2 = 169$
OA = 13 cm

(b) $\sin(\text{angle AOM}) = \frac{12}{13}$
Angle AOM = 67.38°, so angle AOB $= 2 \times 67.38 = 134.8°$
Answer: (a) 13 cm (b) 134.8°

3 Simultaneous equations

Solve the simultaneous equations
$-4x - 2y = -36$
$-5x + y = -24$

To match the x terms: (1) × 5 gives $-20x - 10y = -180$, and (2) × 4 gives $-20x + 4y = -96$
Subtract the equations to eliminate x: $-14y = -84$
y = 6
Substitute y = 6 into (1): $-4x - 12 = -36$
$-4x = -24$, so x = 6
Answer: x = 6, y = 6

4 Average speed

Fatima drives 60 km at an average speed of 30 km/h.
The drive then continues for another 160 km at an average speed of 80 km/h.
Work out the average speed for the whole journey.

Time for first part $= 60 \div 30 = 2$ hours
Time for second part $= 160 \div 80 = 2$ hours
Total distance = 220 km, total time = 4 hours
Average speed $= 220 \div 4 = 55$ km/h
Answer: 55 km/h

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5 Column vectors

The vectors $\begin{pmatrix} 2k \\ 18 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3k \end{pmatrix}$ are parallel.
k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same
$\frac{2k}{3} = \frac{18}{3k}$
$2k \times 3k = 18 \times 3$
$6\,k^2 = 54$
$k^2 = 9$
k = 3 (k is positive)
Answer: k = 3

6 HCF and LCM

(a) Write 280 and 1176 as products of prime factors.
(b) Find the HCF and the LCM of 280 and 1176.
Give your answers as numbers.

(a) $280 = 2^3 \times 5 \times 7$ and $1176 = 2^3 \times 3 \times 7^2$
(b) HCF $= 2^3 \times 7 = 56$
LCM $= 2^3 \times 3 \times 5 \times 7^2 = 5880$
Answer: (a) $2^3 \times 5 \times 7$; $2^3 \times 3 \times 7^2$ (b) HCF = 56, LCM = 5880

7 Rearranging when the subject appears twice

Make u the subject of $v = \frac{4u + 6}{2u - 5}$

Multiply by $2u - 5$: $v(2u - 5) = 4u + 6$
Expand: $2uv - 5v = 4u + 6$
Collect the u terms: $2uv - 4u = 6 + 5v$
Factorise: $u(2v - 4) = 5v + 6$
$u = \frac{5v + 6}{2v - 4}$
Answer: $u = \frac{5v + 6}{2v - 4}$

8 Speed and a moving train

A train travels at a constant speed of 72 km/h.
It takes 25 seconds to cross completely over a bridge which is 210 m long.
Work out the length of the train, in metres.

Speed = 72 km/h = 72,000 ÷ 3600 = 20 m/s
Distance travelled in 25 s = 20 × 25 = 500 m
This is the length of the bridge plus the length of the train
Train = 500 − 210 = 290 m
Answer: 290 m

9 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 41° and angle CAQ = 50°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 50° (alternate segment theorem)
(b) Angle ACB = angle PAB = 41° (alternate segment theorem)
Answer: (a) 50° (b) 41°

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10 Tangent to a circle: proof

Show that the line $y = -x - 12$ is a tangent to the circle $x^2 + y^2 = 72$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -x - 12$ into $x^2 + y^2 = 72$: $x^2 + (-x - 12)^2 = 72$

Expand: $2x^2 + 24x + 72 = 0$

$b^2 - 4ac = 24^2 - 4 \times 2 \times 72 = 576 - 576 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-24}{2 \times 2} = -6$, and $y = -(-6) - 12 = -6$. The point is $(-6, -6)$

Answer: $(-6, -6)$