**Day 126** **S Tier 10-a-day: Model Solutions**

## 1 Solving trigonometric equations

Solve $7 \tan x - 6 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = \frac{6}{7}$

Related acute angle: $\tan^{-1}(\frac{6}{7}) = 40.6°$

tangent is positive in the 1st and 3rd quadrants, so $x = 40.6°$ and $x = 180° + 40.6°$
$x = 40.6°$ and $x = 220.6°$
**Answer: $x = 40.6°$ or $x = 220.6°$**

## 2 Factorising fully

Factorise fully $x^4 - 81$

$x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9)$
$x^2 - 9$ is also a difference of two squares: $(x - 3)(x + 3)$
**Answer: $(x^2 + 9)(x - 3)(x + 3)$**

## 3 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 2 cm and the area of triangle ADE is 48 $cm^2$.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 2 = 6 cm

Length scale factor = $6 \div 4 = \frac{3}{2}$

Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$

Area of triangle ABC = $48 \times \frac{9}{4} = 108$ $cm^2$

Trapezium = $108 - 48 = 60$ $cm^2$
**Answer: 60 $cm^2$**

## 4 Equal roots and real roots

The equation $kx^2 + 6x + 3 = 0$ has equal roots.
Find the value of k.

Equal roots means $b^2 - 4ac = 0$
$(6)^2 - 4 \times k \times 3 = 0$, so $36 = 12k$
$k = 3$
**Answer: $k = 3$**

## 5 Recurring decimals

Write $0.0\dot{5}6\dot{2}$ as a fraction in its simplest form.

Let $x = 0.056256...$
$10x = 0.562562...$
$10{,}000x = 562.562562...$
Subtract: $9990x = 562$
$x = \frac{562}{9990} = \frac{281}{4995}$
**Answer: $\frac{281}{4995}$**

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## 6 Iteration

Use the iteration formula $x_{n+1} = 2 + \frac{8}{x_n}$ with $x_0 = 1$ (Calculator)
to work out $x_1$, $x_2$ and $x_3$.
Give each answer correct to 3 decimal places.

$x_1 = 2 + \frac{8}{1} = 10.00000...$

$x_2 = 2 + \frac{8}{x_1} = 2.80000...$

$x_3 = 2 + \frac{8}{x_2} = 4.85714...$

Rounded: $x_1 = 10.000$, $x_2 = 2.800$, $x_3 = 4.857$

**Answer: $x_1 = 10.000$, $x_2 = 2.800$, $x_3 = 4.857$**

## 7 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 7 : 3.
Then $\frac{1}{2}$ of the blue and $\frac{3}{4}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{1}{2} \times 7 = \frac{7}{2}$ of the blue part, $\frac{1}{4} \times 3 = \frac{3}{4}$ of the green part

$\frac{7}{2} : \frac{3}{4}$

Multiply by 4: 14 : 3

**Answer: 14 : 3**

## 8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 5**a** − **b**, OQ = 14**a** − 13**b** and OR = 26**a** − 29**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 9**a** − 12**b**
QR = OR − OQ = 12**a** − 16**b**

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4

**Answer: (b) 3 : 4**

## 9 Bounds of speed, density and pressure

A runner covers a distance of 320 m, correct to 2 significant figures.
The race takes 65 s, correct to the nearest whole number.
Work out the upper bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

315 ≤ distance < 325; 64.5 ≤ time < 65.5
Upper bound of speed = biggest distance ÷ smallest time = 325 ÷ 64.5
= 5.03876... = 5.04 (3 s.f.)

**Answer: 5.04 m/s**

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## 10 Probability from a frequency tree

In a population of 10,000 people, 4% have a certain disease.
A test gives a positive result for 75% of people who have the disease and for 3% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

4% of 10,000 = 400 have the disease and 9,600 do not
Flagged and genuine: 400 × 75% = 300
Flagged but not: 9,600 × 3% = 288
Total flagged or positive = 300 + 288 = 588

Probability = 300 ÷ 588 = $\frac{25}{49}$

**Answer:** $\mathbf{\frac{25}{49}}$