Day 125 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -3$, $q = -4$ and $r = 4$.

Substitute the values: $(-3 - (-4))^2 - (-3 + (-4))^2$
$= 1 - 49$
$= -48$
Answer: -48

2 Arc length and sector area

A sector of a circle has radius 4 cm.
The length of the arc of the sector is π cm.
(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 4$, so $\frac{\theta}{360} \times 8 = 1$

$\theta = 1 \times \frac{360}{8} = 45°$

(b) Area = $\frac{45}{360} \times \pi \times 4^2 = 2\pi$ cm^2

Answer: (a) 45° (b) 2π cm^2

3 Distance-time graphs

The graph shows Leila's cycle ride to a café and back.
(a) Work out Leila's speed on the way back.
(b) Work out Leila's average speed for the whole ride, including the stop.

(a) Return: 30 km in 1 hours, so speed = 30 ÷ 1 = 30 km/h
(b) Total distance = 2 × 30 = 60 km, total time = 4 hours
Average speed = 60 ÷ 4 = 15 km/h
Answer: (a) 30 km/h (b) 15 km/h

4 Compound interest

Evan puts £1,500 in a savings account.
Compound interest is paid at 6% in year 1, 3% in year 2 and 5% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.06, 1.03, 1.05
£1,500 × 1.06 × 1.03 × 1.05
= £1,719.59
Answer: £1,719.59

5 Combined transformations

The point P(−6, −2) is reflected in the x-axis.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−6, 2)
After the second transformation: P'' = (2, 6)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−y, −x)
The single mapping is (x, y) → (−y, −x)
This is a reflection in the line $y = -x$
Answer: (a) (2, 6) (b) Reflection in the line $y = -x$

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6 Comparing box plots

The box plots show the times taken by two groups, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 80 − 40 = 40
IQR of B = 75 − 65 = 10
(b) Group A has the lower median (65 compared with 70), so on average Group A was faster
Group B has the smaller interquartile range (10 compared with 40), so its times are more consistent
Answer: (a) A: 40, B: 10 (b) A has the lower median, B has the smaller IQR

7 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £35,000 is taxed at 20%.
All income above that is taxed at 45%.
Lena pays £15,325 in income tax in a year.
Work out Lena's income for the year.

Tax on the 20% band = 20% of £35,000 = £7,000
Tax paid at 45% = £15,325 − £7,000 = £8,325
Income taxed at 45% = £8,325 ÷ 0.45 = £18,500
Income = £12,500 + £35,000 + £18,500 = £66,000
Answer: £66,000

8 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to the square of t.
When v = 6 and t = 3, P = 16.
Work out the value of P when v = 6 and t = 2.

$P = \frac{kv^2}{t^2}$

$16 = \frac{k \times 36}{9}$, so $k = 16 \times 9 \div 36 = 4$

$P = \frac{4 \times 36}{4} = 36$

Answer: P = 36

9 Perpendicular bisector

A is the point (2, −3) and B is the point (0, 1).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{2+0}{2}, \frac{-3+1}{2}) = (1, -1)$

Gradient of AB = $\frac{1-(-3)}{0-2} = -2$

The perpendicular gradient is $-1 \div (-2) = \frac{1}{2}$

Using the midpoint: $y + 1 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x - \frac{3}{2}$

Answer: $y = \frac{1}{2}x - \frac{3}{2}$

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10 Bounds of roots

The volume of a cube is 884 cm^3, correct to the nearest cm^3.
Work out the lower bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$883.5 \leq \text{volume} < 884.5$

Lower bound of edge = $\sqrt[3]{883.5}$
= 9.59553... = 9.60 (3 s.f.)
Answer: 9.60 cm