Day 124 **S Tier 10-a-day: Model Solutions**

1 Quadratic formula and surds

Solve $3x^2 + 7x - 3 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 3, b = 7, c = −3

$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-3)}}{2 \times 3} = \frac{-7 \pm \sqrt{85}}{6}$

x = −2.7033... or x = 0.3699...
x = −2.70 or x = 0.37 (2 d.p.)
Answer: x = −2.70 or x = 0.37

2 Capture and recapture

20 deer in a forest are caught, marked and put back.
Later, a second sample of 120 deer is caught. 2 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 2 out of 120

This should match the proportion marked in the whole population: $\frac{2}{120} = \frac{20}{N}$

N = 20 × 120 ÷ 2 = 1200
(b) No deer are born, die or leave between the samples, and the deer mix evenly
Answer: (a) 1200 (b) The population stays the same between samples

3 Equations in disguise

Solve $x - 12\sqrt{x} + 35 = 0$

Let $u = \sqrt{x}$: $u^2 - 12u + 35 = 0$
Factorise: (u − 5)(u − 7) = 0, so u = 5 or u = 7
$\sqrt{x} = 5$ or $\sqrt{x} = 7$
x = 25 or x = 49
Answer: x = 25 or x = 49

4 Fractional and negative indices

Work out $(\frac{81}{16})^{-\frac{1}{4}}$

A negative power means take the reciprocal: $(\frac{81}{16})^{-\frac{1}{4}} = (\frac{16}{81})^{\frac{1}{4}}$

The denominator 4 means the fourth root: the fourth root of $\frac{16}{81}$ is $\frac{2}{3}$

The numerator 1 means raise to the power 1: $(\frac{2}{3})^1 = \frac{2}{3}$

Answer: $\frac{2}{3}$

5 Expanding three brackets

Expand and simplify $(x - 4)^2(x - 1)$

Expand the first two brackets: $(x - 4)^2 = x^2 - 8x + 16$
Multiply by (x − 1): $x(x^2 - 8x + 16) - (x^2 - 8x + 16)$
$= x^3 - 9x^2 + 24x - 16$
Answer: $x^3 - 9x^2 + 24x - 16$

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6 Fraction problems

A barrel is $\frac{1}{5}$ full.

18 more litres of oil are added and the barrel is then $\frac{1}{4}$ full.

Work out the capacity of the barrel.

Fraction added = $\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$

$\frac{1}{20}$ of the capacity is 18 litres

Capacity = $18 \div \frac{1}{20} = 360$

Answer: 360 litres

7 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 30°

$\cos 30° = \frac{\sqrt{3}}{2}$

$x = 12 \times \cos 30° = 12 \times \frac{\sqrt{3}}{2}$

$x = 6\sqrt{3}$

Answer: $x = 6\sqrt{3}$

8 Percentages

A is 25% more than R.
R is 5% less than C.
Work out A as a percentage of C.

A = 1.25 × R and R = 0.95 × C
A = 1.25 × 0.95 × C = 1.1875 × C
1.1875 × 100 = 118.75%
Answer: 118.75%

9 Loci and regions

ABCD is a rectangle with AB = 12 cm and AD = 14 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 6 cm from A
AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{6}{12}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 6 cm, height $12 \sin 60° = 6\sqrt{3}$ cm, area $= \frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 12^2 = 12\pi$

Area = $18\sqrt{3} + 12\pi$ cm^2

Answer: $(18\sqrt{3} + 12\pi)$ cm^2

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10 Finding the rate of interest

Evan invested £17,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £18,928.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $17500 \times m^2 = 18928.00$

$m^2 = 18928.00 \div 17500 = 1.0816$

$m = \sqrt{1.0816} = 1.04$

Rate = $(1.04 - 1) \times 100 = 4\%$

Answer: 4% per year