Day 123 **S Tier 10-a-day: Model Solutions**

1 Surds and fractions

Simplify $\frac{3}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$

Give your answer with a rational denominator.

Common denominator: $(2+\sqrt{2})(2-\sqrt{2}) = 4 - 2 = 2$

Numerator: $3(2-\sqrt{2}) - (2+\sqrt{2})$

$= 4 - 4\sqrt{2}$

$\frac{4-4\sqrt{2}}{2} = 2 - 2\sqrt{2}$

Answer: $2 - 2\sqrt{2}$

2 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{5}$.

Events on different occasions are independent.

Work out the smallest number of days needed so that the probability of this happening at least once is greater than 0.75. (Calculator)

P(at least once in n days) = $1 - \frac{4}{5}^n$

n = 6: $1 - \frac{4}{5}^6 = 0.7379$

n = 7: $1 - \frac{4}{5}^7 = 0.7903$

0.7903 is the first value above 0.75

Answer: 7

3 Simplifying surds

Write $\sqrt{18} - \sqrt{32} + \sqrt{50}$ in the form $a\sqrt{2}$ where a is an integer.

$\sqrt{18} = 3\sqrt{2}$; $\sqrt{32} = 4\sqrt{2}$; $\sqrt{50} = 5\sqrt{2}$

$(3 - 4 + 5)\sqrt{2} = 4\sqrt{2}$

Answer: $4\sqrt{2}$

4 Completing the square

(a) Write $x^2 - 4x - 4$ in the form $(x + p)^2 + q$

(b) Write down the coordinates of the turning point of the curve $y = x^2 - 4x - 4$

(a) Half of -4 is -2, so $x^2 - 4x = (x - 2)^2 - 4$

$x^2 - 4x - 4 = (x - 2)^2 - 4 - 4$

$= (x - 2)^2 - 8$

(b) The minimum is when $x - 2 = 0$, so $x = 2$ and $y = -8$

Answer: (a) $(x - 2)^2 - 8$ (b) $(2, -8)$

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = $(5x - 15)°$, angle ACB = $(2x + 36)°$ and angle CAQ = 58°.

(a) Work out the value of x.

(b) Work out the size of angle BAC.

The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)

$2x + 36 = 5x - 15$

$3x = 51$, so $x = 17$

(b) Angle PAB = 70°

Angles on the straight line PAQ: angle BAC = $180 - 70 - 58 = 52°$

Answer: (a) x = 17 (b) 52°

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6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (3, 7).
The curve crosses the y-axis at (0, −11).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (3, 7), so $y = a(x - 3)^2 + 7$
When x = 0, y = −11: $a \times 9 + 7 = -11$, so $a = -2$
$y = -2(x - 3)^2 + 7$
Expand: $y = -2x^2 + 12x - 11$
Answer: $y = -2x^2 + 12x - 11$

7 Histograms

The histogram shows the masses of parcels delivered by a courier.
(a) Work out the total number of parcels.
(b) Estimate the number of parcels with a mass between 35 kg and 45 kg.

Frequency = frequency density × class width
20 to 40: 1.2 × 20 = 24
40 to 50: 0.6 × 10 = 6
50 to 60: 0.8 × 10 = 8
60 to 90: 0.4 × 30 = 12
(a) Total = 50
(b) 35 to 40: 1.2 × 5 = 6
40 to 45: 0.6 × 5 = 3, so estimate = 9
Answer: (a) 50 (b) 9

8 Transformations of graphs

The graph of y = f(x) has a minimum point at (−2, 6).
Write down the coordinates of the turning point of the graph of
(a) y = 3f(x)
(b) y = 2f(−x)

(a) Start with the turning point (−2, 6)
Apply the transformation to the point: (−2, 18)
(b) f(−x) changes the x-coordinate to 2, and 2f(x) changes the y-coordinate to 12
Turning point: (2, 12)
Answer: (a) (−2, 18) (b) (2, 12)

9 Finding the rate of interest

Grace invested £16,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £18,662.40.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $16000 \times m^2 = 18662.40$
$m^2 = 18662.40 \div 16000 = 1.1664$
$m = \sqrt{1.1664} = 1.08$
Rate = (1.08 − 1) × 100 = 8%
Answer: 8% per year

10 Algebraic proof (harder)

Show algebraically that the difference between the squares of any two odd numbers is a multiple of 8.

Let the odd numbers be 2a + 1 and 2b + 1, where a and b are integers.
$(2a + 1)^2 - (2b + 1)^2 = 4a^2 + 4a - 4b^2 - 4b = 4(a - b)(a + b + 1)$
(a − b) + (a + b + 1) = 2a + 1 is odd, so one of the two brackets is even
So (a − b)(a + b + 1) is even and 4(a − b)(a + b + 1) is a multiple of 8
Answer: Proved