Day 122 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 10:55 (London time).
It takes 5 hours 10 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 10:55 + 5 h 10 min = 16:05
Singapore is 7 hours ahead of London, so add 7 hours
23:05
Answer: 23:05 the same day

2 Comparing costs

FitLife has a joining fee of £1 plus £6 per month.
GymNow has a joining fee of £19 plus £1.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 10 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $1 + 6n = 19 + 1.50n$
$4.50n = 18$, so $n = 4$
(b) FitLife: £1 + £6 × 10 = £61
GymNow: £19 + £1.50 × 10 = £34
GymNow is cheaper by £27
Answer: (a) 4 (b) GymNow, by £27

3 Speed-time graphs

The graph shows the speed of a cyclist during 20 seconds.
(a) Work out the acceleration during the first 5 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = $(14 - 4) \div 5 = 2$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{4 + 14}{2} \times 5 = 45$; rectangle: $14 \times 12 = 168$; triangle: $\frac{1}{2} \times 3 \times 14 = 21$
Total = 234 m, so average speed = 234 ÷ 20 = 11.7 m/s
Answer: (a) 2 m/s^2 (b) 11.7 m/s

4 Laws of indices

Given that $\frac{25^5 \times 5^3}{25^3} = 5^n$,
find the value of n.

$25 = 5^2$
Numerator: $5^{10} \times 5^3 = 5^{13}$
Denominator: 5^6
Subtract the powers: $13 - 6 = 7$
Answer: n = 7

5 Mean of a changing set

The mean of 11 numbers is 5.
One number is removed and the mean of the remaining 10 numbers is 1.8.
Work out the number that was removed.

Total of the 11 numbers = 11 × 5 = 55
Total of the 10 remaining = 10 × 1.8 = 18
Number removed = 55 − 18 = 37
Answer: 37

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6 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 9}{x^2 - 2x - 15}$

Factorise the top: $(x - 3)(x + 3)$
Factorise the bottom: $(x + 3)(x - 5)$
Cancel the common factor $(x + 3)$: $\frac{x - 3}{x - 5}$

Answer: $\mathbf{\frac{x - 3}{x - 5}}$

7 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x - 2)$ cm, $(2x - 5)$ cm and $(x + 7)$ cm. The longest side is $(x + 7)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x - 2)^2 + (2x - 5)^2 = (x + 7)^2$
Expand and collect terms: $4x^2 - 38x - 20 = 0$
Factorise: $2(x - 10)(2x + 1) = 0$
$x = 10$ or $x = -\frac{1}{2}$, and x must be positive, so $x = 10$
(b) The two shorter sides are 8 cm and 15 cm
Area $= \frac{1}{2} \times 8 \times 15 = 60$ cm^2

Answer: (a) x = 10 (b) 60 cm^2

8 Cone made from a sector

A sector of a circle has radius 12 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector $= \frac{180}{360} \times 2\pi \times 12 = 12\pi$ cm
This is the circumference of the base: $2\pi r = 12\pi$, so $r = 6$ cm
(b) Slant height = 12 cm, so height$^2 = 12^2 - 6^2 = 108$, height $= 6\sqrt{3}$ cm
Volume $= \frac{1}{3} \times \pi \times 6^2 \times 6\sqrt{3} = 72\sqrt{3}\pi$ cm^3

Answer: (a) 6 cm (b) $72\sqrt{3}\pi$ cm^3

9 Recurring decimals

Convert the recurring decimal $0.2\dot{4}\dot{0}$ to a fraction.
Give your fraction in its simplest form.

Let x = 0.2404...
10x = 2.4040...
1000x = 240.4040...
Subtract: 990x = 238
$x = \frac{238}{990} = \frac{119}{495}$

Answer: $\mathbf{\frac{119}{495}}$

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10 Algebraic proof (harder)

Prove that n(n + 1)(n + 2)(n + 3) + 1 is a square number for all positive integer values of n.

$n(n + 3) = n^2 + 3n$ and $(n + 1)(n + 2) = n^2 + 3n + 2$

Let $m = n^2 + 3n$. Then the expression is $m(m + 2) + 1$

$= m^2 + 2m + 1 = (m + 1)^2$

$= (n^2 + 3n + 1)^2$, which is the square of an integer

Answer: Proved