Day 121 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 31 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 14 from Year 7. Work out the size of this sample.

Total = 70 + 30 + 120 + 90 = 310

(a) Year 7: $\frac{70}{310} \times 31 = 7$

(b) Fraction sampled = $14 \div 70 = \frac{1}{5}$, so sample size = $\frac{1}{5} \times 310 = 62$

Answer: (a) 7 (b) 62

2 Changing the subject

Make x the subject of $y = \frac{6}{x^2}$

where x is positive.

Multiply by x^2: $yx^2 = 6$

Divide by y: $x^2 = \frac{6}{y}$

Square root: $x = \sqrt{\frac{6}{y}}$

Answer: $x = \sqrt{\frac{6}{y}}$

3 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(4x - 20)°$
Angle PQR = $(2x + 5)°$
Angle PRS = $(4x + 35)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$4x + 35 = 6x - 15$
$2x = 50$
$x = 25$
(b) Angle PRS = 135° so angle PRQ = 180° − 135° = 45°
Answer: (a) x = 25 (b) 45°

4 Exchange rates

Priya changes £600 into dollars at £1 = $1.18.
Priya spends $580 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Priya get back?

£600 × 1.18 = $708
708 − 580 = $128
128 ÷ 1.25 = £102.40
Answer: £102.40

5 Squares, cubes and prime factors

Find the smallest positive integer k such that 162 × k is a square number.

$162 = 2 \times 3^4$
For a square number every power must be a multiple of 2
Multiply by 2 to get $2^2 \times 3^4$
k = 2
Answer: k = 2

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6 Enlargement

Triangle P has vertices A(1, −1), B(0, 2) and C(0, 1).
Triangle Q has vertices A'(−8, −7), B'(−6, −13) and C'(−6, −11).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −1, x-step from A' to B' = 2
Scale factor = 2 ÷ −1 = −2
The centre C satisfies: C + −2(A − C) = A', so (1 − −2)C = A' − −2 A
x: 3 × x = (−8) − (−2) × 1 = −6, so x = −2
y: 3 × y = (−7) − (−2) × (−1) = −9, so y = −3
Answer: Enlargement, scale factor −2, centre (−2, −3)

7 Completing the square

(a) Write $x^2 - 4x + 2$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 4x + 2$

(a) Half of −4 is −2, so $x^2 - 4x = (x - 2)^2 - 4$
$x^2 - 4x + 2 = (x - 2)^2 - 4 + 2$
$= (x - 2)^2 - 2$
(b) The minimum is when $x - 2 = 0$, so x = 2 and y = −2
Answer: (a) $(x - 2)^2 - 2$ (b) (2, −2)

8 Equal roots and real roots

The equation $2x^2 + 10x + k = 0$ has two different real solutions.
Find the range of possible values of k.

Two different real solutions means $b^2 - 4ac > 0$
$10^2 - 4 \times 2 \times k > 0$, so $100 - 8k > 0$
$100 > 8k$, so $k < \frac{25}{2}$
Answer: $k < \frac{25}{2}$

9 Successive percentage change

The value of a car is decreased by 25%.
The new amount is then decreased by 50%.
Work out the overall percentage change.

Multipliers: 0.75 and 0.5
0.75 × 0.5 = 0.375
0.375 is a decrease of 62.5%
Answer: 62.5% decrease

10 Nested surds

Show that $\sqrt{70 + 30\sqrt{5}} = 5 + 3\sqrt{5}$

Square the right-hand side: $(5 + 3\sqrt{5})^2 = 25 + 30\sqrt{5} + 45$
$= 70 + 30\sqrt{5}$
So $\sqrt{70 + 30\sqrt{5}} = 5 + 3\sqrt{5}$
Answer: $\sqrt{70 + 30\sqrt{5}} = 5 + 3\sqrt{5}$