Day 120 **S Tier 10-a-day: Model Solutions**

1 Laws of indices

Given that $\frac{16 \times 8}{4^3} = 2^n$,

find the value of n.

$16 = 2^4$, $8 = 2^3$, $4 = 2^2$

Numerator: $2^4 \times 2^3 = 2^7$

Denominator: 2^6

Subtract the powers: $7 - 6 = 1$

Answer: n = 1

2 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)

(a) Work out the length of AB, marked x, to 1 decimal place.

(b) Work out the area of triangle ABC to 3 significant figures.

The diagram is not drawn accurately.

(a) $x^2 = 14^2 + 8^2 - 2 \times 14 \times 8 \times \cos 70°$

$x^2 = 183.39$

x = 13.5 cm (1 d.p.)

(b) Area = $\frac{1}{2} \times 14 \times 8 \times \sin 70° = 52.6$ cm^2 (3 s.f.)

Answer: (a) 13.5 cm (b) 52.6 cm^2

3 LCM problems

A church bell rings every 6 minutes.

Another bell rings every 20 minutes.

A third bell rings every 24 minutes.

They all ring together at 8:00 am.

How many times do they all ring together between 8:00 am and 3:00 pm, including both 8:00 am and 3:00 pm if they do?

Find the LCM of 6, 20 and 24

$6 = 2 \times 3$, $20 = 2^2 \times 5$, $24 = 2^3 \times 3$

LCM = $2^3 \times 3 \times 5 = 120$

Total time from 8:00 am to 3:00 pm = 420 minutes

$420 \div 120 = 3.5$, so there are 3 more times after the start

Including the start: 3 + 1 = 4

Answer: 4

4 Area of a triangle using sine

Triangle ABC has AB = 16 cm and BC = 8 cm.

The area of triangle ABC is $32\sqrt{3}$ cm^2.

Angle ABC is obtuse.

Work out the size of angle ABC.

Do not use a calculator.

Area = $\frac{1}{2} \times$ AB $\times$ BC $\times \sin$ B

$32\sqrt{3} = \frac{1}{2} \times 16 \times 8 \times \sin$ B

$\sin B = 32\sqrt{3} \div 64 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is obtuse, so $B = 180° - 60° = 120°$

Answer: 120°

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5 Quadratic formula and surds

Solve $2x^2 + 9x + 1 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 2, b = 9, c = 1

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$$x = \frac{-9 \pm \sqrt{73}}{4}$$

Answer: $\mathbf{x = \frac{-9 \pm \sqrt{73}}{4}}$

6 Relative speed

Nadia and Tariq live 130 km apart.
At 10:00 they set off towards each other on the same road.
Nadia drives at 45 km/h and Tariq drives at 85 km/h.
(a) At what time do they meet?
(b) How far from Nadia's home do they meet?

Closing speed = 45 + 85 = 130 km/h
(a) Time = 130 ÷ 130 = 1 hours = 60 minutes, so they meet at 11:00
(b) Distance = 45 × 1 = 45 km
Answer: (a) 11:00 (b) 45 km

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (2, 27) and (4, 243).
Find the value of k and the value of a.

Substitute the points: $k\,a^2 = 27$ and $k\,a^4 = 243$

Divide: $a^2 = \frac{243}{27} = 9$

a = 3 (positive, since a > 0)
From the first point: k × 9 = 27, so k = 3
Answer: k = 3, a = 3

8 Perpendicular bisector

A is the point (6, −5) and B is the point (−2, −3).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{6 + (-2)}{2}, \frac{-5 + (-3)}{2}) = (2, -4)$

Gradient of AB = $\frac{-3 - (-5)}{-2 - 6} = -\frac{1}{4}$

The perpendicular gradient is $-1 \div (-\frac{1}{4}) = 4$

Using the midpoint: y + 4 = 4(x − 2)
Rearrange: −4x + y = −12
Multiply by −1: 4x − y = 12
Answer: 4x − y = 12

9 Powers of standard form

Work out $(7 \times 10^1)^2$
Give your answer in standard form.

$(7 \times 10^1)^2 = 7^2 \times (10^1)^2$
$7^2 = 49$ and $(10^1)^2 = 10^2$
$49 \times 10^2 = 4.9 \times 10^3$
Answer: $\mathbf{4.9 \times 10^3}$

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10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 47.2.
Work out the value of x.

Mid-points: 20, 40, 60, 80
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1760 + 60x) \div (40 + x) = 47.2$
$1760 + 60x = 47.2(40 + x) = 1888 + 47.2x$
$(60 - 47.2)x = 1888 - 1760$
$12.8x = 128$, so $x = 10$
Answer: x = 10