Day 12 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 160°.
Work out the number of sides of the polygon.

Exterior angle = 180° − 160° = 20°
Number of sides = 360° ÷ 20° = 18
Answer: 18

2 Reverse percentages

The number of members of a club is decreased by 40% in one year.
The next year the number of members is decreased by 20%.
There are now 432 members.
How many members were there to begin with?

Multipliers: 0.6, 0.8
Overall multiplier = 0.6 × 0.8 = 0.48
Original = final ÷ multiplier = 432 ÷ 0.48
= 900
Answer: 900

3 Simultaneous equations

Solve the simultaneous equations
$3x - 4y = 28$
$-x + 2y = -12$

To match the x terms: (2) × 3 gives $-3x + 6y = -36$
Add the equations to eliminate x: $2y = -8$
$y = -4$
Substitute $y = -4$ into (1): $3x + 16 = 28$
$3x = 12$, so $x = 4$
Answer: $x = 4$, $y = -4$

4 HCF and LCM

The HCF of two numbers is 3 and their LCM is 42.
One of the numbers is 6.
Work out the other number.

HCF × LCM = product of the two numbers
3 × 42 = 126
Other number = 126 ÷ 6 = 21
Answer: 21

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
5, 12, 27, 50, 81
Find an expression for the nth term.

First differences: 7, 15, 23, 31
Second differences: 8, 8, 8, so $a = 8 \div 2 = 4$
Subtract $4n^2$ from each term: 1, −4, −9, −14, −19
The remaining linear sequence has nth term $-5n + 6$
nth term = $4n^2 - 5n + 6$
Answer: $4n^2 - 5n + 6$

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6 Upper and lower bounds

a = 70 and b = 48,
both correct to the nearest whole number.
Work out the lower bound of a + b.

Each value is rounded, so it can be up to 0.5 above or below
$69.5 \leq a < 70.5$; $47.5 \leq b < 48.5$
Lower bound = 69.5 + 47.5 = 117
Answer: 117

7 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Thu is 20.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Fri.

(a) (23 + x + 10) ÷ 3 = 20
23 + x + 10 = 60, so x = 27
(b) (27 + 10 + 14) ÷ 3 = 51 ÷ 3 = 17
Answer: (a) x = 27 (b) 17

8 Equations with algebraic fractions

Solve $\frac{6}{x-5} + \frac{6}{x+5} = \frac{1}{2}$

Multiply every term by $(x-5)(x+5)$: $6(x+5) + 6(x-5) = \frac{1}{2} \times (x-5)(x+5)$

Multiply by 2 to clear the fraction: $12(x+5) + 12(x-5) = (x-5)(x+5)$

Expand and rearrange so that one side is 0: $x^2 - 24x - 25 = 0$
Factorise: $(x-25)(x+1) = 0$
$x = -1$ or $x = 25$
Answer: $x = -1$ or $x = 25$

9 Vectors: intersecting lines

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 1 : 4. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of **a** and **b**.
(b) Find OX in terms of **a** and **b**.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{5}s\mathbf{a} + (1-s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1-t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$

Equate: $\frac{1}{5}s = 1 - t$ and $1 - s = \frac{1}{2}t$

Solving: $s = \frac{5}{9}$ and $t = \frac{8}{9}$

OX = $\frac{1}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$

Answer: (a) $\frac{1}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$

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10 Probability from a frequency tree

A factory makes 20,000 parts. 10% of them are faulty.
A scanner flags 80% of faulty parts and wrongly flags 10% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

10% of 20,000 = 2,000 are faulty and 18,000 are not
Flagged and genuine: 2,000 × 80% = 1,600
Flagged but not: 18,000 × 10% = 1,800
Total flagged or positive = 1,600 + 1,800 = 3,400

Probability = 1,600 ÷ 3,400 = $\frac{8}{17}$

Answer: $\mathbf{\frac{8}{17}}$