Day 119 **S Tier 10-a-day: Model Solutions**

1 Standard form

Work out $(4.2 \times 10^4) \times (1.5 \times 10^{-7})$
Give your answer in standard form.

$4.2 \times 1.5 = 6.3$
$10^4 \times 10^{-7} = 10^{-3}$
$= 6.3 \times 10^{-3}$
Answer: 6.3×10^{-3}

2 Factorising fully

Factorise fully $2x^3 - 72x$

Take out the common factor 2x: $2x(x^2 - 36)$
$x^2 - 36$ is a difference of two squares: $(x - 6)(x + 6)$
Answer: $2x(x - 6)(x + 6)$

3 Laws of indices

Given that $\frac{16^4 \times 4^3}{8} = 2^n$,
find the value of n.

$16 = 2^4$, $4 = 2^2$, $8 = 2^3$
Numerator: $2^{16} \times 2^6 = 2^{22}$
Denominator: 2^3
Subtract the powers: $22 - 3 = 19$
Answer: n = 19

4 Distance-time graphs

The graph shows Liam's cycle ride to a café and back.
(a) Work out Liam's speed on the way back.
(b) Work out Liam's average speed for the whole ride, including the stop.

(a) Return: 30 km in 2.5 hours, so speed = 30 ÷ 2.5 = 12 km/h
(b) Total distance = 2 × 30 = 60 km, total time = 5 hours
Average speed = 60 ÷ 5 = 12 km/h
Answer: (a) 12 km/h (b) 12 km/h

5 Forming and solving quadratic equations

A rectangle has length (2x + 6) cm and width (x + 5) cm.
The area of the rectangle is 160 cm^2.
(a) Show that $2x^2 + 16x - 130 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(2x + 6)(x + 5) = 160$, so $2x^2 + 16x + 30 = 160$
$2x^2 + 16x - 130 = 0$
(b) $2x^2 + 16x - 130 = 0$: $2(x - 5)(x + 13) = 0$
$x = 5$ ($x = -13$ is not possible as lengths are positive)
Length = 16 cm and width = 10 cm, so perimeter = 2 × (16 + 10) = 52 cm
Answer: (b) 52 cm

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6 Combined transformations

The point P(−2, −6) is reflected in the line y = −x.
The image is then reflected in the x-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (6, 2)
After the second transformation: P'' = (6, −2)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, −x) → (−y, x)
The single mapping is (x, y) → (−y, x)
This is a rotation 90° anticlockwise about the origin
Answer: (a) (6, −2) (b) Rotation 90° anticlockwise about the origin

7 Tree diagrams without replacement

A bag contains 3 orange cubes, 2 purple cubes and 3 pink cubes.
Two cubes are taken at random without replacement.
Work out the probability that (a) at least one is orange, (b) both are the same colour.

(a) P(at least one orange) = 1 − P(none) = $1 - \frac{5}{8}\times\frac{4}{7} = \frac{9}{14}$

(b) Add the ways: $\frac{3}{8}\times\frac{2}{7} + \frac{2}{8}\times\frac{1}{7} + \frac{3}{8}\times\frac{2}{7} = \frac{1}{4}$

Answer: (a) $\frac{9}{14}$ (b) $\frac{1}{4}$

8 Counting factors

$N = 2^4 \times 3^2 \times 7^2$
How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 4, that is 4 choices
Power of 3: 0 to 2, that is 3 choices
Power of 7: 0 to 2, that is 3 choices
Total = 4 × 3 × 3 = 36
Answer: 36

9 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 2 : 5.
Then 14 more apples are added and 13 more pears are added.
The ratio is now 2 : 3.
How many apples were there at the start?

Let the numbers at the start be 2k and 5k
(2k + 14) : (5k + 13) = 2 : 3, so 3(2k + 14) = 2(5k + 13)
6k + 42 = 10k + 26
−4k = −16, so k = 4
Number of apples at the start: 2 × 4 = 8
Answer: 8

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on OB with ON : NB = 3 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{3}\mathbf{a} = \frac{1}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{4}t\mathbf{b}$

Equate: $\frac{1}{3}s = 1 - t$ and $1 - s = \frac{3}{4}t$

Solving: $s = \frac{1}{3}$ and $t = \frac{8}{9}$

OX = $\frac{1}{9}\mathbf{a} + \frac{2}{3}\mathbf{b}$

Answer: (a) $\frac{1}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{9}\mathbf{a} + \frac{2}{3}\mathbf{b}$