Day 118 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+7}{2} + \frac{x+1}{6} = 7$

Multiply every term by 6: $3(x + 7) + (x + 1) = 42$
Expand: $4x + 22 = 42$
$4x = 20$
$x = 5$
Answer: x = 5

2 Mean of a changing set

The mean of 6 numbers is 60.5.
One number is removed and the mean of the remaining 5 numbers is 67.4.
Work out the number that was removed.

Total of the 6 numbers = 6 × 60.5 = 363
Total of the 5 remaining = 5 × 67.4 = 337
Number removed = 363 − 337 = 26
Answer: 26

3 Tests of divisibility

The 4-digit number 660k is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 6 + 6 + 0 + k = 12 + k
This must be a multiple of 9, so k = 6
Answer: k = 6

4 Ratio and expressions

x : y = 4 : 5
Work out the ratio (4x − 2y) : (6x + 1y)
Give your answer in its simplest form.

x : y = 4 : 5, so let x = 4t and y = 5t
4(4t) − 2(5t) = 6t
6(4t) + 1(5t) = 29t
6t : 29t = 6 : 29
Answer: 6 : 29

5 Reverse percentages

The value of a house is decreased by 8% in one year.
The next year its value is decreased by 10%.
It is now worth £372,600.
Work out the value of the house before the first change.

Multipliers: 0.92, 0.9
Overall multiplier = 0.92 × 0.9 = 0.828
Original = final ÷ multiplier = £372,600 ÷ 0.828
= £450,000
Answer: £450,000

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6 Reverse percentage growth and decay

A car loses 20% of its value each year.
After 3 years it is worth £3,072.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.8

Original × 0.8^3 = £3,072

Original = £3,072 ÷ 0.8^3 = £6,000

Answer: £6,000

7 Algebraic fractions: adding and subtracting

Write $\frac{2}{x-6} + \frac{6}{x+3}$ as a single fraction.
Simplify the numerator.

The common denominator is $(x - 6)(x + 3)$

$$\frac{2}{x-6} + \frac{6}{x+3} = \frac{2(x+3) + 6(x-6)}{(x-6)(x+3)}$$

Expand the numerator: $8x - 30$

$$\frac{8x-30}{(x-6)(x+3)}$$

Answer: $\mathbf{\frac{8x-30}{(x-6)(x+3)}}$

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 4**a** + 3**b**, OQ = 6**a** and OR = 10**a** − 6**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 2**a** − 3**b**

QR = OR − OQ = 4**a** − 6**b**

QR = 2 PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 1 : 2

Answer: (b) 1 : 2

9 Tangent to a circle: proof

Show that the line $y = -3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -3x - 10$ into $x^2 + y^2 = 10$: $x^2 + (-3x - 10)^2 = 10$

Expand: $10x^2 + 60x + 90 = 0$

$b^2 - 4ac = 60^2 - 4 \times 10 \times 90 = 3600 - 3600 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-60}{2 \times 10} = -3$, and $y = -3 \times (-3) - 10 = -1$. The point is $(-3, -1)$

Answer: $(-3, -1)$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on OB with ON : NB = 3 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{5}t\mathbf{b}$

Equate: $\frac{1}{5}s = 1 - t$ and $1 - s = \frac{3}{5}t$

Solving: $s = \frac{5}{11}$ and $t = \frac{10}{11}$

OX = $\frac{1}{11}\mathbf{a} + \frac{6}{11}\mathbf{b}$

Answer: (a) $\frac{1}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{11}\mathbf{a} + \frac{6}{11}\mathbf{b}$