**Day 117** **S Tier 10-a-day: Model Solutions**

## 1 Averages with algebra

Here are five numbers: $3x + 2$, $x + 1$, $x + 5$, $x + 4$, $3x - 1$.
The mean of the five numbers is 13.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(3x + 2) + (x + 1) + (x + 5) + (x + 4) + (3x - 1) = 13 \times 5 = 65$
$9x + 11 = 65$, so $9x = 54$ and $x = 6$
The numbers are 20, 7, 11, 10, 17
(b) In order: 7, 10, 11, 17, 20, so the median is 11
**Answer: (a) x = 6 (b) 11**

## 2 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 0$, $\cos x° = 1$, so $a + c = -3$
At $x = 180$, $\cos x° = -1$, so $-a + c = -9$
Add: $2c = -12$, so $c = -6$
Subtract: $2a = 6$, so $a = 3$
**Answer: a = 3, c = −6**

## 3 Inverse proportion in context

9 workers can build a wall in 20 days.
After 6 days, 6 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $9 \times 20 = 180$ worker-days
Work done = $9 \times 6 = 54$, so work left = 126
Workers left = $9 - 6 = 3$
Time = $126 \div 3 = 42$ days
**Answer: 42 days**

## 4 Laws of indices

Given that $\frac{3^4 \times 27^2}{3^3} = 3^n$,

find the value of n.

$27 = 3^3$
Numerator: $3^4 \times 3^6 = 3^{10}$
Denominator: $3^3$
Subtract the powers: $10 - 3 = 7$
**Answer: n = 7**

## 5 Three-part ratios

Callum, Dina and Maya share some sweets.
Callum : Dina = 9 : 3
Dina : Maya = 6 : 3
(a) Work out the ratio Callum : Dina : Maya.
(b) They share 99 sweets altogether. How many sweets does Dina get?

(a) Make the Dina part the same: 9 : 3 = 18 : 6 and 6 : 3 = 6 : 3
Callum : Dina : Maya = 18 : 6 : 3 = 6 : 2 : 1
(b) Total parts = 9, so 1 part = $99 \div 9 = 11$
Dina gets $2 \times 11 = 22$ sweets
**Answer: (a) 6 : 2 : 1 (b) 22 sweets**

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## 6 Simplifying algebraic fractions

Simplify fully $\frac{x^2 + 7x + 10}{x^2 - 2x - 8}$

Factorise the top: $(x + 2)(x + 5)$
Factorise the bottom: $(x + 2)(x - 4)$
Cancel the common factor $(x + 2)$: $\frac{x + 5}{x - 4}$

**Answer:** $\frac{x + 5}{x - 4}$

## 7 Shaded areas: exact answers

ABCD is a square of side 20 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of $\pi$.

Area of quarter circle (centre A, radius 20) = $\frac{1}{4} \times \pi \times 20^2 = 100\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = $2 \times 100\pi - 400 = 200\pi - 400$
**Answer:** $(200\pi - 400)\ \text{cm}^2$

## 8 Growth and decay

A van is worth £1,200 when new.
Its value decreases by 8% each year.
After how many whole years will it first be worth less than £800? (Calculator)

Multiplier each year = 0.92
After 4 years: £1,200 × $0.92^4$ = £859.67
After 5 years: £1,200 × $0.92^5$ = £790.90
So the first time it is less than £800 is after 5 years
**Answer: 5 years**

## 9 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 60°

$\cos 60° = \frac{1}{2}$

$x = 6\sqrt{3} \times \cos 60° = 6\sqrt{3} \times \frac{1}{2}$

$x = 3\sqrt{3}$

**Answer:** $x = 3\sqrt{3}$

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## 10 Inverse functions

$f(x) = \frac{2x + 4}{2x - 4}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(2)$.

(a) Let $y = \frac{2x + 4}{2x - 4}$. Then $y(2x - 4) = 2x + 4$

$2xy - 4y = 2x + 4$

Collect x terms: $2xy - 2x = 4 + 4y$

Factorise: $x(2y - 2) = 4y + 4$, so $x = \frac{4y + 4}{2y - 2}$

$f^{-1}(x) = \frac{4x + 4}{2x - 2}$

(b) $f^{-1}(2) = \frac{4 \times 2 + 4}{2 \times 2 - 2} = \frac{12}{2} = 6$

**Answer: (a) $f^{-1}(x) = \frac{4x + 4}{2x - 2}$ (b) 6**