Day 116 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 22°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 22°
Angle COB is an exterior angle of triangle OAC: 22 + 22 = 44°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 44°

2 Inequalities

(a) Solve $-2 - 6x \leq 10$
(b) n is an integer. Write down all the values of n that satisfy
$4 \leq 3n + 1 \leq 13$

(a) Add 2: $-6x \leq 12$
Divide by -6 and reverse the inequality sign: $x \geq -2$
(b) Subtract 1: $3 \leq 3n \leq 12$
Divide by 3: $1 \leq n \leq 4$
Integers: 1, 2, 3, 4
Answer: (a) $x \geq -2$ (b) 1, 2, 3, 4

3 Laws of indices

Given that $\frac{27^2 \times 3^4}{9^2} = 3^n$,

find the value of n.

$27 = 3^3$, $9 = 3^2$
Numerator: $3^6 \times 3^4 = 3^{10}$
Denominator: 3^4
Subtract the powers: $10 - 4 = 6$
Answer: n = 6

4 Moving averages

The table shows the monthly sales (thousands) over six periods.
The 3-point moving average centred on Apr is 18.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on May.

(a) $(11 + x + 22) \div 3 = 18$
$11 + x + 22 = 54$, so $x = 21$
(b) $(21 + 22 + 32) \div 3 = 75 \div 3 = 25$
Answer: (a) x = 21 (b) 25

5 Upper and lower bounds

a = 8.8 and b = 1.3,
both correct to 1 decimal place.
Work out the lower bound of a − b.

Each value is rounded, so it can be up to 0.05 above or below
$8.75 \leq a < 8.85$; $1.25 \leq b < 1.35$
Smallest answer: smallest a minus biggest b = 8.75 − 1.35 = 7.4
Answer: 7.4

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6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 33° and angle CAQ = 56°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 56° (alternate segment theorem)
(b) Angle ACB = angle PAB = 33° (alternate segment theorem)
Answer: (a) 56° (b) 33°

7 Histograms

The histogram shows the times of people spent waiting at a clinic.
(a) Work out the total number of people.
(b) Estimate the number of people with a time between 5 minutes and 15 minutes.

Frequency = frequency density × class width
0 to 10: 0.8 × 10 = 8
10 to 20: 1.2 × 10 = 12
20 to 40: 0.6 × 20 = 12
40 to 60: 1 × 20 = 20
60 to 70: 1 × 10 = 10
(a) Total = 62
(b) 5 to 10: 0.8 × 5 = 4
10 to 15: 1.2 × 5 = 6, so estimate = 10
Answer: (a) 62 (b) 10

8 Equations with algebraic fractions

Solve $\frac{2}{x-1} + \frac{6}{x+5} = \frac{1}{2}$

Multiply every term by $(x-1)(x+5)$: $2(x+5) + 6(x-1) = \frac{1}{2} \times (x-1)(x+5)$
Multiply by 2 to clear the fraction: $4(x+5) + 12(x-1) = (x-1)(x+5)$
Expand and rearrange so that one side is 0: $x^2 - 12x - 13 = 0$
Factorise: $(x-13)(x+1) = 0$
$x = -1$ or $x = 13$
Answer: $x = -1$ or $x = 13$

9 At least one

The probability that the 8:15 train is late on any day is $\frac{3}{20}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 5 days.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{3}{20})^5 = \frac{17}{20}^5 = 0.4437053125$
P(at least once) = 1 − P(never) = 1 − 0.4437053125
= 0.556
Answer: 0.556

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10 Functions that are their own inverse

$f(x) = \frac{5x - 5}{3x - 5}$ where $x \neq \frac{5}{3}$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(3x - 5)$

$ff(x) = \frac{5(5x - 5) + (-5)(3x - 5)}{3(5x - 5) - 5(3x - 5)}$

Top $= 25x - 25 - 15x + 25 = 10x$

Bottom $= 15x - 15 - 15x + 25 = 10$

$ff(x) = \frac{10x}{10} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{5x - 5}{3x - 5}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{5x - 5}{3x - 5}}$