Day 115 **S Tier 10-a-day: Model Solutions**

1 Arithmetic sequences

The 5th term of an arithmetic sequence is 24.
The 10th term of the sequence is 39.
(a) Find an expression for the nth term of the sequence.
(b) Is 81 a term of the sequence? You must show your working.

(a) Common difference = $(39 - 24) \div (10 - 5) = 3$
nth term = $3n + c$. For n = 5: $3 \times 5 + c = 24$, so $c = 9$
nth term = $3n + 9$
(b) Solve $3n + 9 = 81$: $3n = 72$, so $n = 24$
n is a positive whole number, so 81 is the 24th term
Answer: (a) 3n + 9 (b) Yes, it is the 24th term

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.
$\frac{4}{5}$, $\frac{5}{11}$, $\frac{3}{11}$, 0.82

Convert each to a decimal: $\frac{4}{5} = 0.8$, $\frac{5}{11} = 0.455$, $\frac{3}{11} = 0.273$, $0.82 = 0.82$

In order: $\frac{3}{11}$, $\frac{5}{11}$, $\frac{4}{5}$, 0.82

Answer: $\frac{3}{11}$, $\frac{5}{11}$, $\frac{4}{5}$, 0.82

3 Arc length and sector area

A sector of a circle has angle 150° and area 15π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{150}{360} \times \pi \times r^2$, so $\frac{150}{360} \times r^2 = 15$

$r^2 = 15 \times \frac{360}{150} = 36$

r = 6 cm

(b) Arc length = $\frac{150}{360} \times 2 \times \pi \times 6 = 5\pi$ cm

Answer: (a) 6 cm (b) 5π cm

4 Forming and solving quadratic equations

The product of two positive numbers is 72.
The larger number is 1 more than the smaller number.
Let the smaller number be n.
(a) Show that $n^2 + n - 72 = 0$
(b) Work out the two numbers.

(a) The numbers are n and n + 1, so $n(n + 1) = 72$
$n^2 + n = 72$, so $n^2 + n - 72 = 0$
(b) Factorise: $(n - 8)(n + 9) = 0$
$n = 8$ or $n = -9$. n is positive, so $n = 8$
The numbers are 8 and 9
Answer: (b) 8 and 9

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5 Laws of indices

Given that $\frac{27^4 \times 9}{9^2} = 3^n$,

find the value of n.

$27 = 3^3$, $9 = 3^2$

Numerator: $3^{12} \times 3^2 = 3^{14}$

Denominator: 3^4

Subtract the powers: $14 - 4 = 10$

Answer: n = 10

6 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)

(a) Work out angle ACB, marked θ, to 1 decimal place.

(b) Work out the area of triangle ABC to 3 significant figures.

The diagram is not drawn accurately.

(a) $\cos\theta = \frac{10^2 + 6^2 - 10^2}{2 \times 10 \times 6} = \frac{36}{120}$

$\theta = 72.5°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 10 \times 6 \times \sin 72.5°$

$= 28.6\ \text{cm}^2$ (3 s.f.)

Answer: (a) 72.5° (b) 28.6 cm^2

7 Histograms

The histogram shows the ages of visitors to a museum.

(a) Work out the total number of visitors.

(b) Estimate the number of visitors with an age between 35 years and 65 years.

Frequency = frequency density × class width

0 to 20: $1.6 \times 20 = 32$

20 to 30: $1.4 \times 10 = 14$

30 to 40: $0.6 \times 10 = 6$

40 to 70: $1.4 \times 30 = 42$

(a) Total = 94

(b) 35 to 40: $0.6 \times 5 = 3$

40 to 65: $1.4 \times 25 = 35$, so estimate = 38

Answer: (a) 94 (b) 38

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.

The point P(3, 4) lies on the circle.

(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.

(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{4}{3} = -\frac{3}{4}$

$y - 4 = (-\frac{3}{4})(x - 3)$

Multiply out and rearrange: $3x + 4y = 25$

(b) On the x-axis y = 0, so $3x = 25$

$x = \frac{25}{3}$, so T is $(\frac{25}{3}, 0)$

Answer: (a) 3x + 4y = 25 (b) T = $(\frac{25}{3}, 0)$

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9 Bounds and accuracy

a = 0.898 and b = 0.234, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.8975 \leq a < 0.8985$; $0.2335 \leq b < 0.2345$
Lower bound = 0.8975 × 0.2335 = 0.20956625
Upper bound = 0.8985 × 0.2345 = 0.21069825
Both bounds round to 0.21 to 2 significant figures, but not to 3 significant figures
Answer: 0.21 (2 s.f.)

10 Inverse proportion

P is inversely proportional to t.
When t = 4, P = 2.
Find the positive value of t when P = 1.

$P = \frac{k}{t}$

$2 = \frac{k}{4}$, so $k = 2 \times 4 = 8$

$1 = \frac{8}{t}$, so $t = \frac{8}{1} = 8$

Answer: t = 8