Day 114 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

20 fish in a lake are caught, tagged and put back.
Later, a second sample of 20 fish is caught. 4 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 4 out of 20

This should match the proportion tagged in the whole population: $\frac{4}{20} = \frac{20}{N}$

N = 20 × 20 ÷ 4 = 100

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 100 (b) The population stays the same between samples

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (3x − 20)°
Angle ABC = (3x − 17)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(3x − 20) + 2(3x − 17) = 180
9x − 54 = 180, so 9x = 234
x = 26
(b) Angle BAC = 3 × 26 − 20 = 58°
Answer: (a) x = 26 (b) 58°

3 Quadratic graphs

The curve $y = 2x^2 - 20x + 32$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0. $2x^2 - 20x + 32 = 0$, so $2(x^2 + 10x + 16) = 0$, giving $2(x - 2)(x - 8) = 0$
x − 2 = 0 or x − 8 = 0, so x = 2 or x = 8
(b) The turning point is halfway between A and B: x = (2 + 8) ÷ 2 = 5
When x = 5: $y = 2 \times 5^2 - 20 \times 5 + 32 = -18$
Answer: (a) A(2, 0), B(8, 0) (b) (5, −18)

4 Fractions

Work out $3\frac{1}{4} \times 2\frac{5}{6} - 2\frac{1}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{13}{4} \times \frac{17}{6} = \frac{221}{24}$

$\frac{221}{24} - \frac{7}{3} = \frac{55}{8}$

$= 6\frac{7}{8}$

Answer: $6\frac{7}{8}$

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5 Similar shapes

Two shapes are mathematically similar.
The smaller has height 16 cm and area 36 cm^2.
The larger has area 225 cm^2.
Work out the height of the larger shape.

Ratio of areas = 225 : 36 = 25 : 4
Ratio of lengths = 5 : 2
Height = $16 \times \frac{5}{2} = 40$ cm

Answer: 40 cm

6 Bounds of speed, density and pressure

A train travels a distance of 1500 km, correct to 2 significant figures.
The journey takes 3.7 hours, correct to 1 decimal place.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

1450 ≤ distance < 1550; 3.65 ≤ time < 3.75
Lower bound of speed = smallest distance ÷ biggest time = 1450 ÷ 3.75
= 386.667... = 387 (3 s.f.)
Answer: 387 km/h

7 Histograms

The histogram shows the ages of visitors to a museum.
(a) Work out the total number of visitors.
(b) Estimate the number of visitors with an age between 25 years and 35 years.

Frequency = frequency density × class width
20 to 30: 0.4 × 10 = 4
30 to 40: 1 × 10 = 10
40 to 60: 0.8 × 20 = 16
60 to 80: 1.4 × 20 = 28
80 to 90: 0.4 × 10 = 4
(a) Total = 62
(b) 25 to 30: 0.4 × 5 = 2
30 to 35: 1 × 5 = 5, so estimate = 7
Answer: (a) 62 (b) 7

8 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 2, 27, 12, 32
Mid-points: 15, 27.5, 42.5, 60
Σfm = 2×15 + 27×27.5 + 12×42.5 + 32×60 = 3202.5, Σf = 73
Mean = 3202.5 ÷ 73 = 43.9
Answer: 43.9

9 Proof with indices

Simplify fully $\frac{7^{m+2} - 7^m}{48}$

$7^{m+2} - 7^m = 7^m(7^2 - 1) = 7^m \times 48$

Divide by 48: $\frac{48 \times 7^m}{48} = 7^m$

Answer: 7^m

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10 Algebraic proof (harder)

Prove that the difference between the squares of any two odd numbers is a multiple of 8.

Let the odd numbers be 2a + 1 and 2b + 1, where a and b are integers.

$(2a + 1)^2 - (2b + 1)^2 = 4a^2 + 4a - 4b^2 - 4b = 4(a - b)(a + b + 1)$

$(a - b) + (a + b + 1) = 2a + 1$ is odd, so one of the two brackets is even

So $(a - b)(a + b + 1)$ is even and $4(a - b)(a + b + 1)$ is a multiple of 8

Answer: Proved