Day 113 **S Tier 10-a-day: Model Solutions**

1 Pressure

A concrete slab in the shape of a cuboid weighs 240 N.
Its smallest face has area 150 cm^2. Its largest face has area 250 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 250 cm^2 = 0.025 m^2
(a) Greatest pressure is on the smallest face: 240 ÷ 0.015 = 16,000 N/m^2
(b) Least pressure is on the largest face: 240 ÷ 0.025 = 9,600 N/m^2
Answer: (a) 16,000 N/m^2 (b) 9,600 N/m^2

2 Standard form

Work out $(5 \times 10^{-5}) \div (1.25 \times 10^{7})$
Give your answer in standard form.

5 ÷ 1.25 = 4
$10^{-5} \div 10^{7} = 10^{-12}$
$= 4 \times 10^{-12}$
Answer: 4×10^{-12}

3 Rearranging when the subject appears twice

Make t the subject of $T = \frac{3t + 4}{t - 6}$

Multiply by $t - 6$: $T(t - 6) = 3t + 4$
Expand: $tT - 6T = 3t + 4$
Collect the t terms: $tT - 3t = 6T + 4$
Factorise: $t(T - 3) = 6T + 4$
$t = \frac{6T + 4}{T - 3}$
Answer: $t = \frac{6T + 4}{T - 3}$

4 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that the first number is even, work out the probability that the product is greater than 8.

(a) There are 4 × 6 = 24 equally likely outcomes, and 10 of them satisfy: the product is greater than 8
Probability = $\frac{5}{12}$
(b) 12 outcomes satisfy: the first number is even. Of these, 6 also satisfy: the product is greater than 8
Probability = $\frac{1}{2}$
Answer: (a) $\frac{5}{12}$ (b) $\frac{1}{2}$

5 Exact trigonometric values

Work out the exact value of cos 60° × tan 30°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 60° \times \tan 30° = \frac{1}{2} \times \frac{\sqrt{3}}{3}$
$= \frac{\sqrt{3}}{6}$
Answer: $\frac{\sqrt{3}}{6}$

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6 Compound interest

Evan invests £11,000 at 4% per year compound interest.
After how many whole years will the investment first be worth more than £15,600?
(Calculator)

Multiplier = 1.04

After 8 years: $11000 \times 1.04^8 = 15054.26$

After 9 years: $11000 \times 1.04^9 = 15656.43$

15656.43 is the first value above £15,600

Answer: 9 years

7 Fractions of fractions

Sofia read $\frac{7}{8}$ of a book on Saturday.

On Sunday she read $\frac{2}{5}$ of the pages that were left.

She still had 42 pages to read.
How many pages are in the book?

After the first part, $\frac{1}{8}$ remains

After the second, $\frac{3}{5}$ of $\frac{1}{8}$ remains = $\frac{3}{40}$ of the total

$\frac{3}{40}$ of the total is 42

Total = $42 \div \frac{3}{40} = 560$

Answer: 560 pages

8 Simple and compound interest

Ravi invests £3,000 for 3 years at 4% per year simple interest.
Hana invests £3,000 for 3 years at 4% per year compound interest.
Work out how much more interest Hana earns than Ravi.
Give your answer to the nearest penny. (Calculator)

Simple interest: £3,000 × 4% × 3 = £360

Compound: $£3,000 \times 1.04^3 = £3,374.59$

Compound interest = £3,374.59 − £3,000 = £374.59

Difference = £374.59 − £360 = £14.59

Answer: £14.59

9 Shaded areas: exact answers

ABCD is a square of side 10 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 10) = $\frac{1}{4} \times \pi \times 10^2 = 25\pi$

The two quarter circles overlap in the shaded region and together cover the whole square

Shaded area = $2 \times 25\pi - 100 = 50\pi - 100$

Answer: $(50\pi - 100)$ cm^2

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10 Tangent to a circle

A circle has equation $(x + 2)^2 + (y - 4)^2 = 85$.
The point P(4, 11) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is $(-2, 4)$. The gradient of the radius to P is $\frac{11 - 4}{4 - (-2)} = \frac{7}{6}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{6}{7}$. Using P: $y - 11 = -\frac{6}{7}(x - 4)$

Multiply by 7: $7(y - 11) = -6(x - 4)$
Expand and rearrange: $6x + 7y = 101$
Answer: 6x + 7y = 101