Day 112 **S Tier 10-a-day: Model Solutions**

1 Quadratic formula and surds

Solve $5x^2 - 8x - 6 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 5, b = −8, c = −6

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 5 \times (-6)}}{2 \times 5} = \frac{8 \pm \sqrt{184}}{10}$$

x = −0.5565... or x = 2.1565...
x = −0.56 or x = 2.16 (2 d.p.)
Answer: x = −0.56 or x = 2.16

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (5x − 10)°
Angle PQR = (3x − 5)°
Angle PRS = (6x + 17)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
6x + 17 = 8x − 15
2x = 32
x = 16
(b) Angle PRS = 113° so angle PRQ = 180° − 113° = 67°
Answer: (a) x = 16 (b) 67°

3 Transformations of graphs

The point P(4, 2) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x + 4) (b) y = f(−x)

(a) y = f(x + 4) is a translation of 4 to the left: (0, 2)
(b) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (−4, 2)
Answer: (a) (0, 2) (b) (−4, 2)

4 Percentage change

The rent of a flat is increased by 400%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 400% increase it is 500.
To return to 100, decrease by 400

$$\text{Percentage decrease} = \frac{400}{500} \times 100 = 80\%$$

Answer: 80%

5 Changing the subject

Make r the subject of $P = \frac{4}{\sqrt{r}}$

Multiply by $\sqrt{r}$: $P\sqrt{r} = 4$

Divide by P: $\sqrt{r} = \frac{4}{P}$

Square both sides: $r = \frac{16}{P^2}$

Answer: $r = \frac{16}{P^2}$

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6 Profit and loss

A retailer buys a games console for £350.
It adds an 80% mark-up to get the selling price.
In a sale the selling price is reduced by 25%.
Work out the percentage profit or loss on the original cost.

Marked price = £350 × 1.8 = £630
Sale price = £630 × 0.75 = £472.50
Profit = £122.50
Percentage = $\frac{122.5}{350} \times 100 = 35\%$

Answer: 35% profit

7 Density of an alloy

An alloy contains 426 g of zinc and 678 g of lead.
The density of zinc is 7.1 g/cm^3. The density of lead is 11.3 g/cm^3.
Work out the density of the alloy.

Volume of zinc = 426 ÷ 7.1 = 60 cm^3
Volume of lead = 678 ÷ 11.3 = 60 cm^3
Total mass = 1,104 g, total volume = 120 cm^3
Density = 1,104 ÷ 120 = 9.2 g/cm^3

Answer: 9.2 g/cm^3

8 Exact lengths and areas

A rectangle has area $(11 + 7\sqrt{3})$ cm^2 and length $(2 + \sqrt{3})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length = $\frac{11 + 7\sqrt{3}}{2 + \sqrt{3}}$

Multiply top and bottom by $(2 - \sqrt{3})$:
Numerator: $(11 + 7\sqrt{3})(2 - \sqrt{3}) = 1 + 3\sqrt{3}$
Denominator: $4 - 3 = 1$
Width = $1 + 3\sqrt{3}$

Answer: $1 + 3\sqrt{3}$ cm

9 Probability and quadratic equations

A bag contains some beads. 7 of them are blue and the rest are black.
Two beads are taken at random without replacement.
The probability that both are blue is $\frac{7}{22}$.
Work out how many beads are black.

Let there be n beads altogether
P(both blue) = $\frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)}$
$\frac{42}{n(n-1)} = \frac{7}{22}$, so $n(n-1) = 132$, giving $n^2 - n - 132 = 0$
$(n - 12)(n + 11) = 0$, so $n = 12$ (n cannot be negative)
Number of beads that are black = $12 - 7 = 5$

Answer: 5

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10 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 8 cm and inner radius 7 cm. (Calculator)
The density of the metal is 2.7 g/cm^3.
The mass of the pipe is 3.82 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 3820 g, so volume = mass ÷ density = 3820 ÷ 2.7 = 1414.8 cm^3
Cross-sectional area = $\pi \times 8^2 - \pi \times 7^2$ = 47.124 cm^2
Length = volume ÷ area = 1414.8 ÷ 47.124 = 30.0 cm (3 s.f.)
Answer: 30.0 cm