Day 111 **S Tier 10-a-day: Model Solutions**

1 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 23.
Work out the value of x.

Mid-points: 5, 15, 25, 35
Estimated mean = $\Sigma fm \div \Sigma f$, so $(515 + 15x) \div (21 + x) = 23$
$515 + 15x = 23(21 + x) = 483 + 23x$
$(15 - 23)x = 483 - 515$
$-8x = -32$, so $x = 4$
Answer: x = 4

2 Changing the subject

Make l the subject of $T = 5\sqrt{l - 3} + 3$

Subtract 3: $T - 3 = 5\sqrt{l - 3}$
Divide by 5: $\frac{T - 3}{5} = \sqrt{l - 3}$
Square both sides: $\frac{(T - 3)^2}{25} = l - 3$
$l = \frac{(T - 3)^2}{25} + 3$
Answer: $\mathbf{l = \frac{(T - 3)^2}{25} + 3}$

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 68°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: $360 - 90 - 90 - 68 = 112°$
(b) The angle at the circumference is half the angle at the centre: $112 \div 2 = 56°$
Answer: (a) 112° (b) 56°

4 Capture and recapture

60 beetles in a field are caught, marked and put back.
Later, a second sample of 80 beetles is caught. 8 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 8 out of 80
This should match the proportion marked in the whole population: $\frac{8}{80} = \frac{60}{N}$
$N = 60 \times 80 \div 8 = 600$
(b) No beetles are born, die or leave between the samples, and the beetles mix evenly
Answer: (a) 600 (b) The population stays the same between samples

5 Expanding brackets with surds

Show that $(5 + 3\sqrt{10})(5 - 3\sqrt{10})$ is an integer.

Use the difference of two squares: $(5 + 3\sqrt{10})(5 - 3\sqrt{10}) = 25 - (3\sqrt{10})^2$
$= 25 - 90 = -65$
Answer: −65

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6 Distance-time graphs

The graph shows Callum's cycle ride to a café and back.
(a) Work out Callum's speed on the way back.
(b) Work out Callum's average speed for the whole ride, including the stop.

(a) Return: 60 km in 2.5 hours, so speed = 60 ÷ 2.5 = 24 km/h
(b) Total distance = 2 × 60 = 120 km, total time = 6 hours
Average speed = 120 ÷ 6 = 20 km/h
Answer: (a) 24 km/h (b) 20 km/h

7 Direct and inverse proportion

P is inversely proportional to the square of t.
When t = 2, P = 1.
(a) Find a formula for P in terms of t.
(b) Find the value of P when t = 1.

(a) $P = \frac{k}{t^2}$

Substitute t = 2 and P = 1: $1 = \frac{k}{2^2}$

k = 4

$P = \frac{4}{t^2}$

(b) $P = \frac{4}{1^2} = 4$

Answer: (a) $P = \frac{4}{t^2}$ (b) 4

8 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 5) cm, DE = 8 cm and BC = 20 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 5} = \frac{8}{20}$
20x = 8(2x + 5)
20x = 16x + 40
4x = 40
x = 10
(b) AB = 2x + 5 = 25 cm
Answer: (a) x = 10 (b) 25 cm

9 Tangency conditions

The line $y = -x + k$ is a tangent to the circle $x^2 + y^2 = 22$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = -x + k$: $x^2 + (-x + k)^2 = 22$
Expand: $2x^2 - 2kx + k^2 - 22 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(-2k)^2 - 4 \times 2 \times (k^2 - 22) = 0$
$-4k^2 + 176 = 0$, so $k^2 = 44$
$k = \pm 2\sqrt{11}$
Answer: $k = \pm 2\sqrt{11}$

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10 Rationalising the denominator

Write $\frac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{7})$:
Numerator: $(5 + \sqrt{7})(3 + \sqrt{7}) = 22 + 8\sqrt{7}$
Denominator: $9 - 7 = 2$
$= 11 + 4\sqrt{7}$
Answer: $11 + 4\sqrt{7}$