Day 110 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 7 numbers is 85.
A new number is added and the mean of all 8 numbers is 74.75.
Work out the new number.

Total of the 7 numbers = 7 × 85 = 595
New total = 8 × 74.75 = 598
New number = 598 − 595 = 3
Answer: 3

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 58°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 58°
Angle COB is an exterior angle of triangle OAC: 58 + 58 = 116°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 116°

3 Percentage change

The rent of a flat is increased by 900%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 900% increase it is 1000.
To return to 100, decrease by 900

Percentage decrease = $\frac{900}{1000} \times 100 = 90\%$

Answer: 90%

4 Arc length and sector area

A sector of a circle has radius 4 cm and angle 240°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{240}{360}$

Arc length = $\frac{240}{360} \times 2 \times \pi \times 4 = \frac{16}{3}\pi$ cm

(b) Area = $\frac{240}{360} \times \pi \times 4^2 = \frac{32}{3}\pi$ cm^2

Answer: (a) $\frac{16}{3}\pi$ cm (b) $\frac{32}{3}\pi$ cm^2

5 Recurring decimals

Write $0.\dot{1}\dot{7}$ as a fraction in its simplest form.

Let x = 0.1717...
100x = 17.1717...
Subtract: 99x = 17
x = $\frac{17}{99}$

Answer: $\frac{17}{99}$

6 Where a line meets the axes

The line L has equation $7x - 24y = 168$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $7x = 168$, so $x = 24$. A is $(24, 0)$
At B, $x = 0$: $-24y = 168$, so $y = -7$. B is $(0, -7)$
(b) $AB^2 = 24^2 + 7^2 = 625$
$AB = \sqrt{625} = 25$
Answer: (a) A(24, 0), B(0, −7) (b) 25

7 Compound interest

Rhys invests £8,500 for 7 years at 2% per year compound interest.
Work out the value of the investment at the end of the 7 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = $1 + 2 \div 100 = 1.02$
$8500 \times 1.02^7 = 9763.83$
To the nearest penny: £9,763.83
Answer: £9,763.83

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 16}{x^2 + x} \div \frac{x^2 - 11x + 28}{x}$

Factorise: $\frac{(x - 4)(x + 4)}{x(x + 1)} \div \frac{(x - 7)(x - 4)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 4)(x + 4)}{x(x + 1)} \times \frac{x}{(x - 7)(x - 4)}$

Cancel $(x - 4)$ and x
$= \frac{(x + 4)}{(x - 7)(x + 1)}$
Answer: $\mathbf{\frac{(x + 4)}{(x - 7)(x + 1)}}$

9 Estimating the median from grouped data

The table gives information about 86 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 9, 25, 55, 76, 86
The median is the value at position $86 \div 2 = 43$
It is in the class 50 to 70, with 25 values before it, so we need 18 more of the 30 values in this class
Median $\approx 50 + \frac{18}{30} \times 20 = 62$
Answer: 62

10 Tangency conditions

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 6$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 6$
Expand: $5x^2 + 4kx + k^2 - 6 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(4k)^2 - 4 \times 5 \times (k^2 - 6) = 0$
$-4k^2 + 120 = 0$, so $k^2 = 30$
$k = \pm\sqrt{30}$
Answer: $\mathbf{k = \pm\sqrt{30}}$